University POLITEHNICA of Bucharest

Applied Mathematics in Optimization Problems

Applications

Assignment Problem

1. **Network Modeling module. Assignment Problem type**
2. **Problems**

1. Network Modeling module. Assignment Problem type

The **Network Modeling** module, **Assignment Problem** type, from the product software **WinQSB**, is used to solve, in a conversational system, the assignment problems. Its main features are the following:

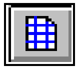
- It accepts the problem data in tabular form with a predefined structure or in graphical form.
- It displays iteration results in tabular or graphical form.
- It displays the solution of the problem in tabular or graphical form.
- It performs *what ... if ...* analysis and parametric

solution analysis.

• The file with the data problem is saved with the **.net** extension.

Running the **Network Modeling** module displays the main window of the module:

Fig.1. The main window of the **Network Modeling** module

The **New Problem** command from the **File** menu or a mouse click on the button  displays the **Net Problem**

Specification dialog box (Fig.2). It contains fields to be filled in/selected by the user.

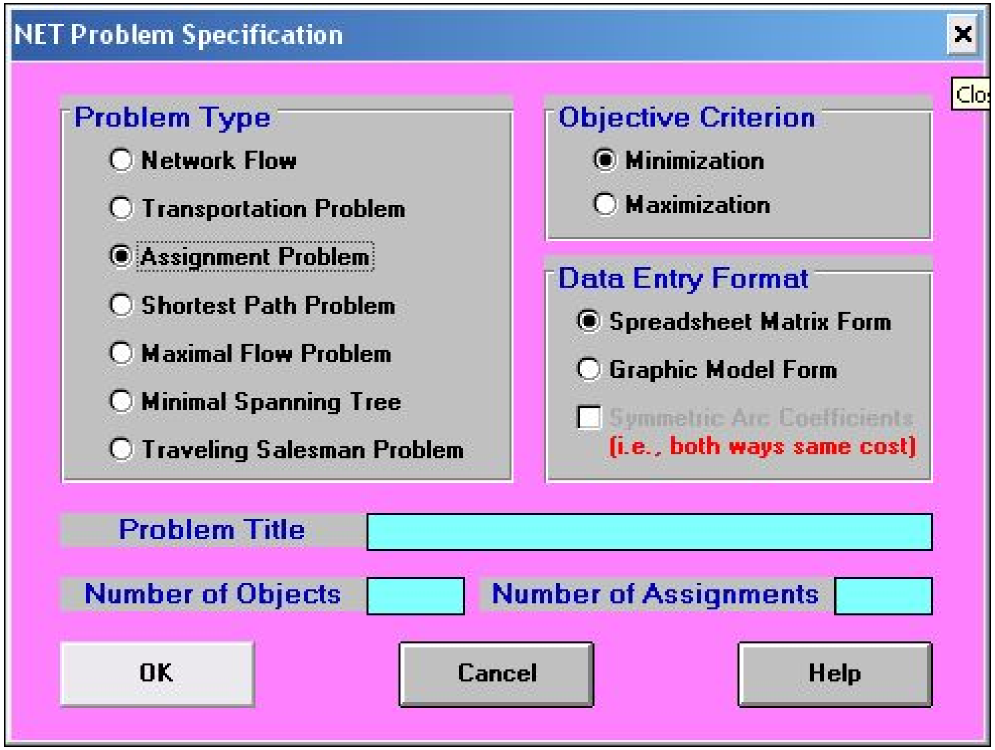


Fig.2. **Net Problem Specification** dialog box

A click on the command button **OK** in the dialog box displays the working window, presented in Fig.3:

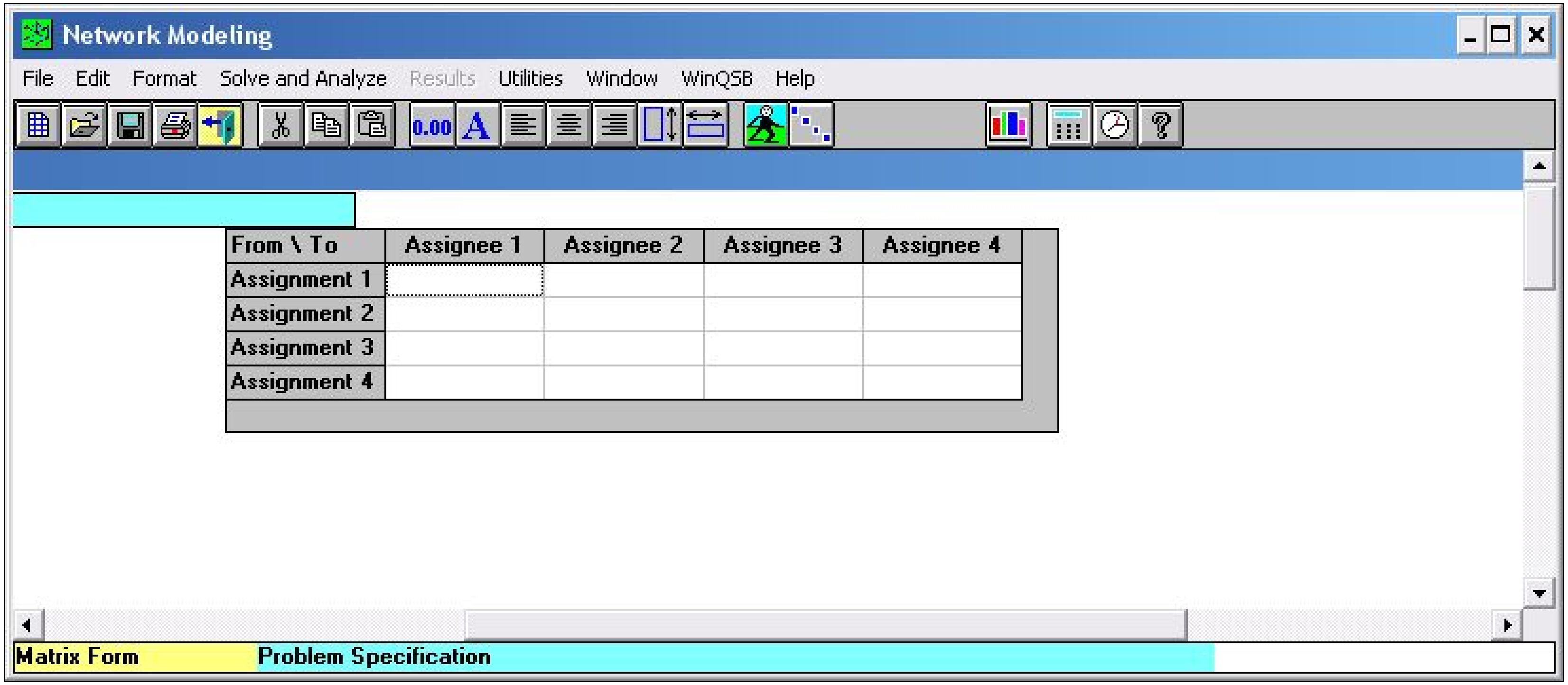


Fig.3. The working window of the module, **Assignment** type

At the top of the working window are:

➢ *Menu bar*, which contains the menus: **File**, **Edit**,

Format, **Solve and Analyze**, **Results**, **Utilities**, **Window**, **WinQSB**, **Help**.

- *Toolbars* for certain submenus.
- *Bar* with the name of the problem.
- *Bar* indicating the current box.

The menus **Files** and **Edit** are illustrated in Fig.4-5.

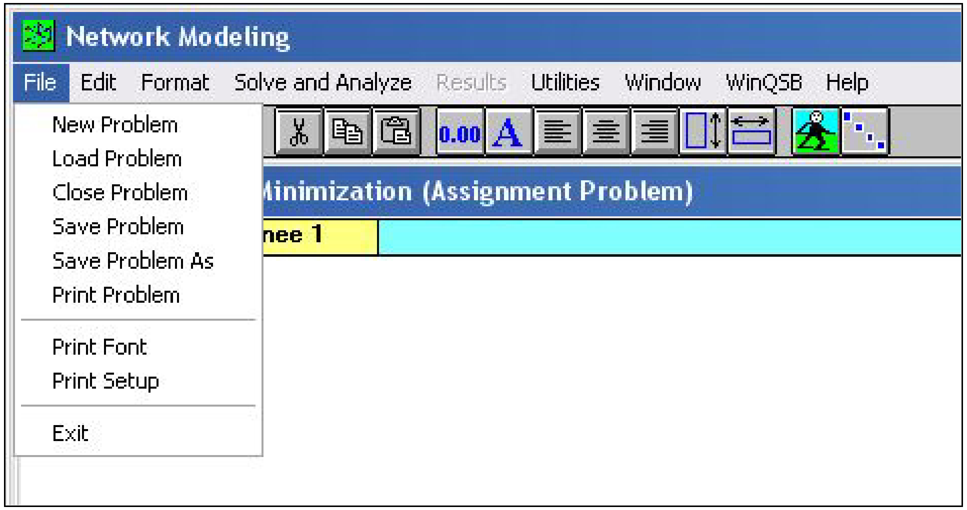


Fig.4. **File** menu

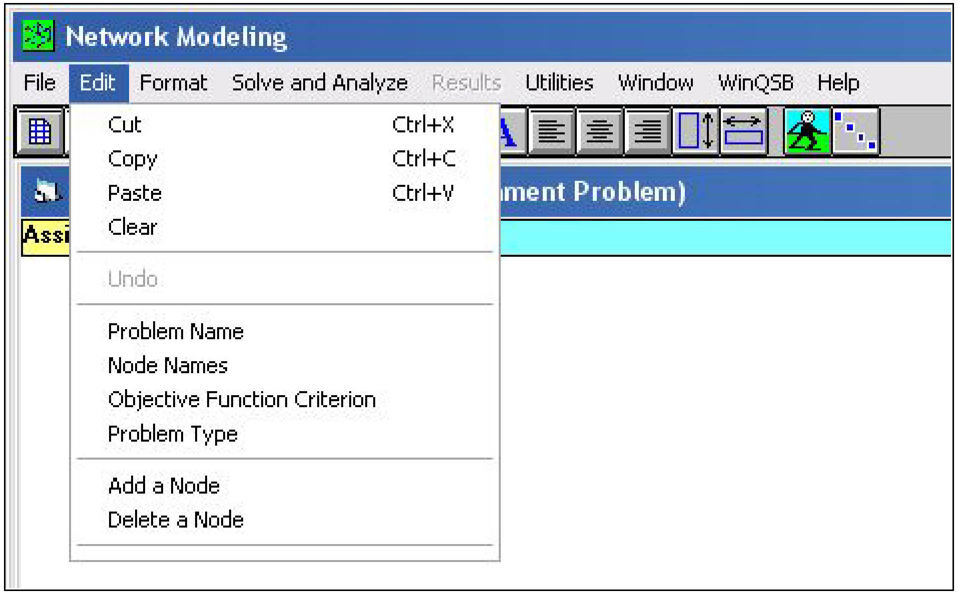


Fig.5. **Edit** menu

With the command **Switch to Graphic Model** from the menu **Format**, it switches from the matrix (tabular) form to the graphic form of the problem. With the command **Switch to Matrix Form**, which appears in the menu **Format**, it returns from the graphic form to the matrix form.

The first three commands from the menu **Solve and Analyze** (Fig.6) are dedicated for solving an assignment problem.

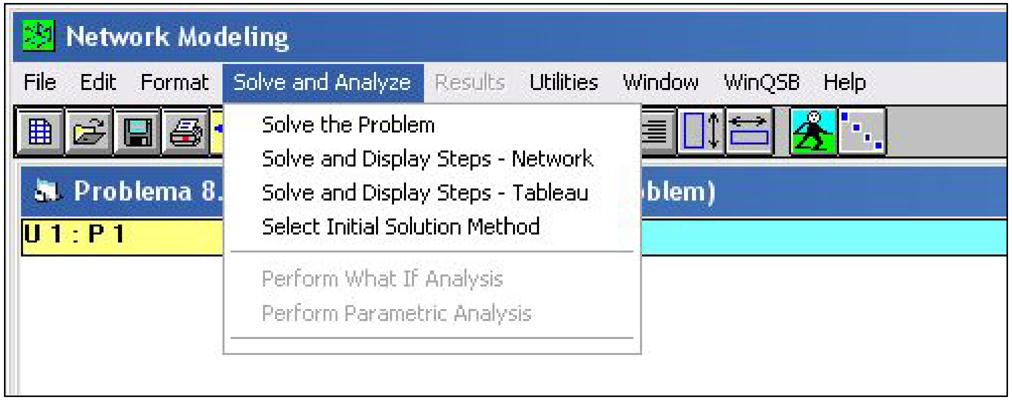


Fig.6. **Solve and Analyze** menu

The menu **Results**, which is displayed after solving the problem, contains the commands presented in Fig.7.

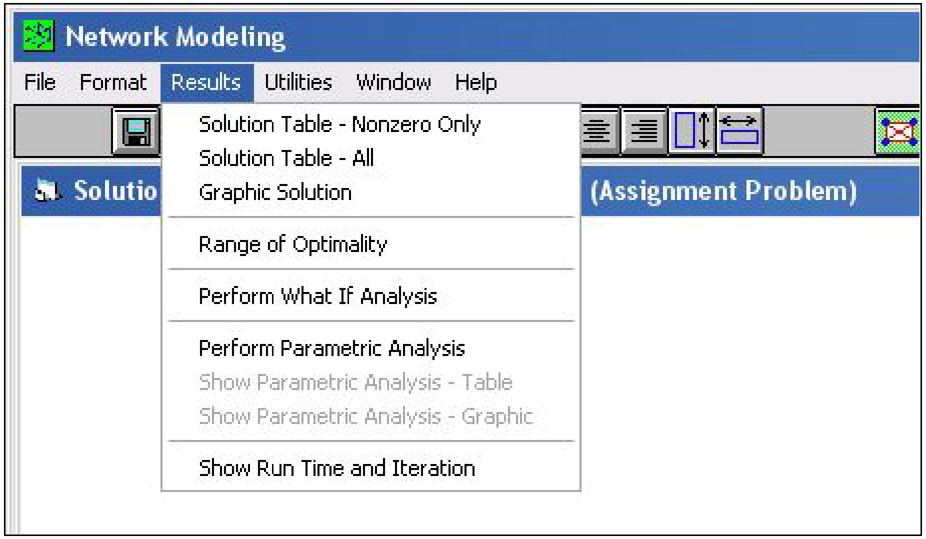


Fig.7. **Results** menu

To solve an assignment problem using the **Network Modeling** module, **Assignment problem** type, the following steps must be completed:

1. Identifying the type of problem.

2. Select the command **New Problem** and complete the fields in the **NET Problem Specification** dialog box.

3. Entering the data of the problem.

4. Save the data with the command **Save Problem As**.

5. Solve the problem with a command from the menu **Solve and Analyze**.

6. Display the obtained results with a command from the menu **Results**.

2. Problems

Problem 1.

The pieces P_1, ..., P_5, whose execution is urgent, can be machined on any of the five machines U_1, ..., U_5. The unit times required for execution (in min.) are presented in the Table 1.

a) Determine the optimal distribution of the pieces on machines so that the total execution time is minimal.

b) Calculate the intervals in which the execution times from the Table 1 may vary so that the previous optimal distribution does not change.

Table 1

	P_1	P_2	P_3	P_4	P_5
U_1	5	9	11	8	4
U_2	9	6	7	6	10
U_3	7	10	6	13	8
U_4	4	6	12	8	5
U_5	8	5	8	7	4

a) The above problem is a minimization assignment problem.

The **Network Modeling** module is called for its solving. It selects the command **New Problem** and it completes the **NET Problem Specification** dialog box as in the Fig.8. Then click the command button **OK**.

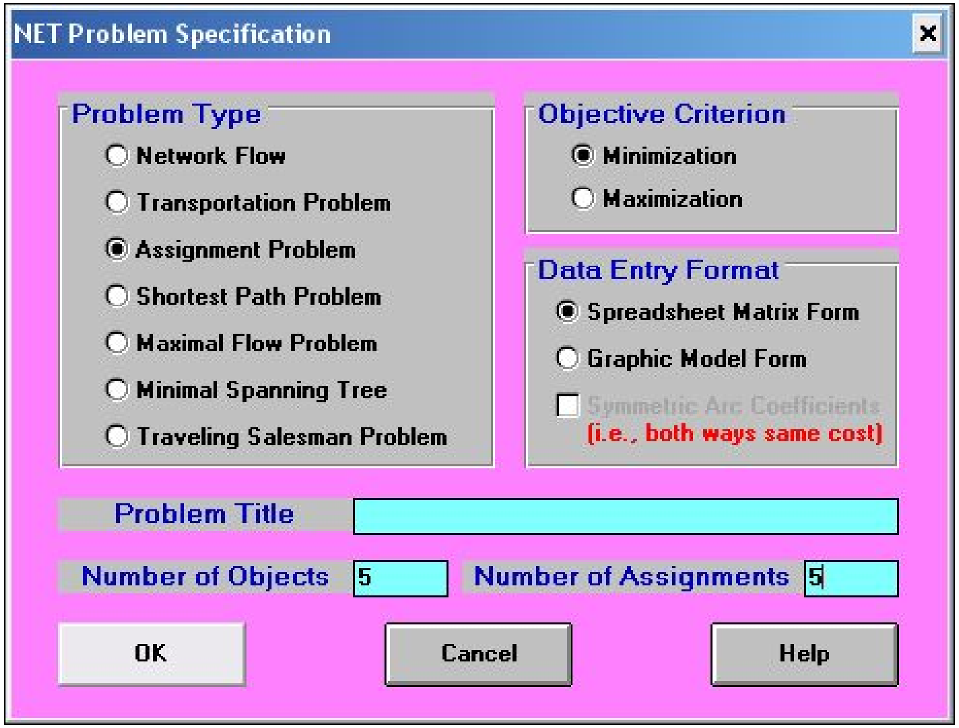


Fig.8. **NET Problem Specification** dialog box

The model of the problem in tabular form is presented in the Fig.9.

The command **Solve the Problem** from the **Solve and Analyze** menu returns the solution of the problem (optimal distribution) in tabular form (Fig.10).

From \ To	P 1	P 2	P 3	P 4	P 5
U 1	5	9	11	8	4
U 2	9	6	7	6	10
U 3	7	10	6	13	8
U 4	4	6	12	8	5
U 5	8	5	8	7	4

Fig.9. The model of the problem in tabular form

05-06-2013	From	To	Assignment	Unit Cost	Total Cost	Reduced Cost
1	U 1	P 5	1	4	4	0
2	U 2	P 4	1	6	6	0
3	U 3	P 3	1	6	6	0
4	U 4	P 1	1	4	4	0
5	U 5	P 2	1	5	5	0
	Total	Objective	Function	Value =	25	

Fig.10. The optimal solution in tabular form

The optimal distribution of the pieces on machines is as follows: U_1 - P_5; U_2 - P_4; U_3 - P_3; U_4 - P_1; U_5 - P_2. The corresponding total execution time is equal to 25 min.

b) With the command **Range of Optimality** from the menu **Results**, it obtains a table (Fig.11) which contains the sensitivity analysis of the optimal solution. So the columns **Allowable Min. Cost** and **Allowable Max. Cost** contain the extremities of some open intervals in which the execution times of the pieces may vary so that the previous solution does not change. Thus $t_{11} \in (3,6)$, $t_{12} \in (7,\infty)$, $t_{13} \in (4,\infty)$ and so on. ▣

05-06-2013 11:42:43	From	To	Unit Cost	Reduced Cost	Basis Status	Allowable Min. Cost	Allowable Max. Cost
1	U 1	P 1	5	0	basic	3	6
2	U 1	P 2	9	2	at bound	7	M
3	U 1	P 3	11	7	at bound	4	M
4	U 1	P 4	8	1	at bound	7	M
5	U 1	P 5	4	0	basic	-2	6
6	U 2	P 1	9	5	at bound	4	M
7	U 2	P 2	6	0	basic	5	9
8	U 2	P 3	7	4	at bound	3	M
9	U 2	P 4	6	0	basic	-3	7
10	U 2	P 5	10	7	at bound	3	M
11	U 3	P 1	7	0	basic	5	8
12	U 3	P 2	10	1	at bound	9	M
13	U 3	P 3	6	0	basic	0	10
14	U 3	P 4	13	4	at bound	9	M
15	U 3	P 5	8	2	at bound	6	M
16	U 4	P 1	4	0	basic	3	6
17	U 4	P 2	6	0	basic	4	7
18	U 4	P 3	12	9	at bound	3	M
19	U 4	P 4	8	2	at bound	6	M
20	U 4	P 5	5	2	at bound	3	M
21	U 5	P 1	8	5	at bound	3	M
22	U 5	P 2	5	0	basic	-M	7
23	U 5	P 3	8	6	at bound	2	M
24	U 5	P 4	7	2	at bound	5	M
25	U 5	P 5	4	2	at bound	2	M

Fig.11. The table with sensitivity analysis of the solution

Problem 2

An express courier company has four means of transport M_1, ..., M_4, which must go to one of the four customers A, B, C, D of the company, located in different areas. The distances (in km.) to be covered to the clients are presented in the Table 2, in which the symbol "-" indicates the impossibility of moving of M_3 towards the client C.

a) Determine the distribution of the means of transport to the four customers so that the total distance traveled to them is minimal.

b) What will be the distribution if customer B gives up the services of the courier company ?

Table 2

	A	*B*	*C*	*D*
M_1	30	25	20	35
M_2	20	15	17	20
M_3	45	22	-	25
M_4	40	35	20	50

a) The above problem is a minimization assignment problem.

The table with the input data is presented in Fig.12.

With the command **Switch to Graphic Model** from the menu **Format**, the graphic model of the problem is obtained, in the form of a bipartite graph (Fig.13).

The command **Solve and Display Steps - Tableau** from the menu **Solve and Analyze** displays the table of

From \ To	A	B	C	D
M1	30	25	20	35
M2	20	15	17	20
M3	45	22	M	25
M4	40	35	20	50

Fig.12. Data of the problem (case a) in tabular form

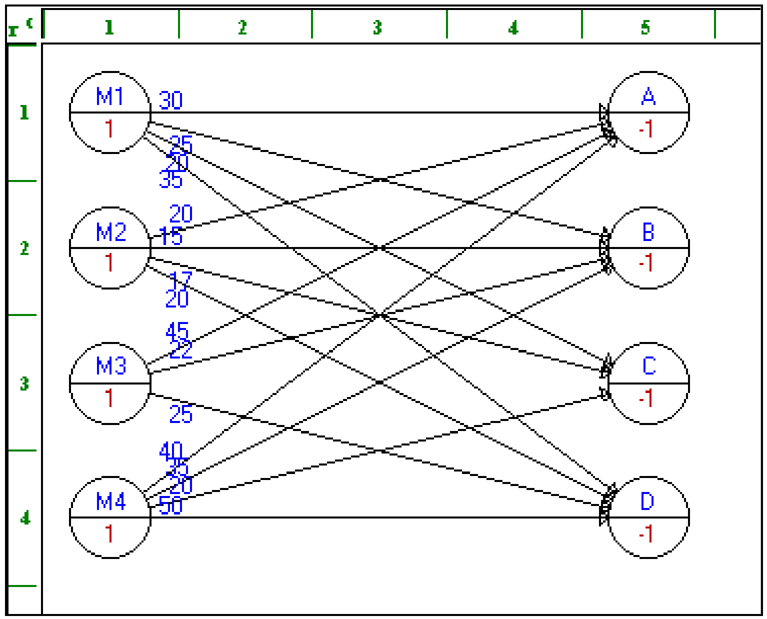


Fig.13. Model of the problem (case a) in graphic form

the first iteration of the solving algorithm (Fig.14).

	A	B	C	D
M1	5	5	0	12
M2	0	0	2	2
M3	10	0	M	0
M4	15	15	0	27

Fig.14. Table of the first iteration of the solving algorithm

The command **Next Iteration** from the menu **Iteration** displays the table of the second (last) iteration of the algorithm (Fig.15).

A new command **Next Iteration** returns the solution of the problem in tabular form (Fig.16).

The command **Graphic Solution** from the menu **Results** displays the solution in graphic form (Fig.17).

	A	B	C	D
M1	0	0	0	7
M2	0	0	7	2
M3	18	0	M	0
M4	10	10	0	22

Fig.15. Table of the last iteration of the solving algorithm

05-06-2013	From	To	Assignment	Unit Cost	Total Cost	Reduced Cost
1	M1	B	1	25	25	0
2	M2	A	1	20	20	0
3	M3	D	1	25	25	0
4	M4	C	1	20	20	0
	Total	Objective	Function	Value =	90	

Fig.16. The solution of the problem (case a) in tabular form

The optimal assignment (distribution) of the means of transport to the four customers is: M_1 - B; M_2 - A; M_3 - D; M_4 - C. The total distance to be covered is equal to 90 km.

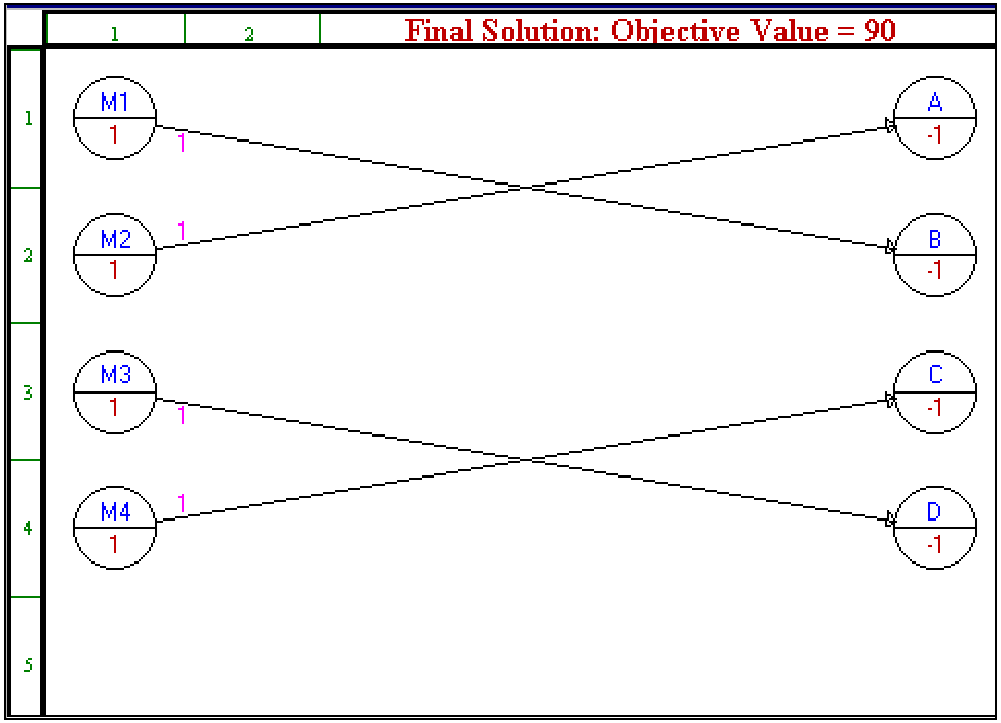


Fig.17. The solution of the problem (case a) in graphic form

b) It deletes the node *B* with the command **Delete a Node** from the menu **Edit**. The graphic model of the problem becomes:

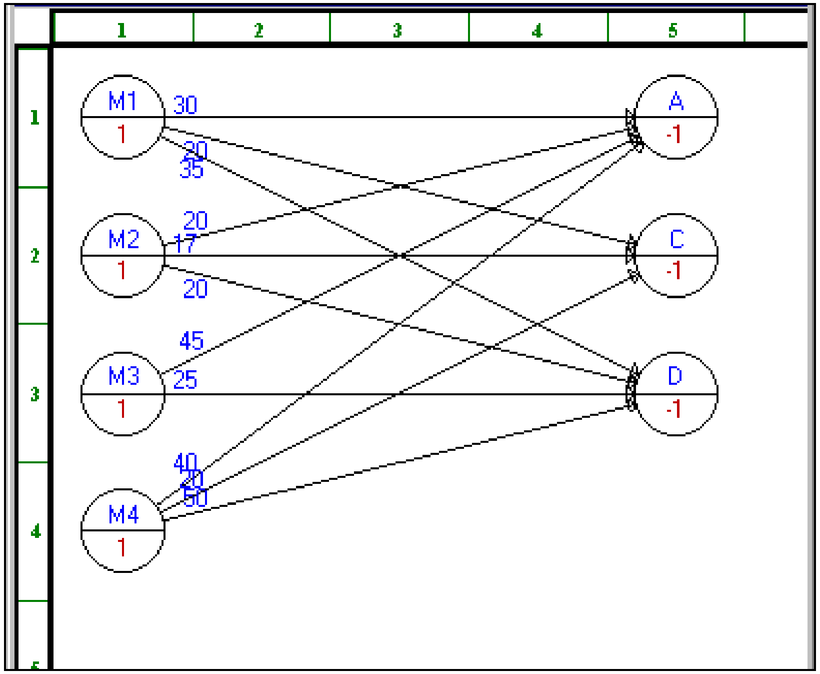


Fig.18. Model of the problem (case b) in graphic form

The solution of the problem in tabular form is obtained with the command **Solve the Problem**. The command **Graphic Solution** displays the solution in graphic form (Fig.19). It is found that the means of transport M_4 is not

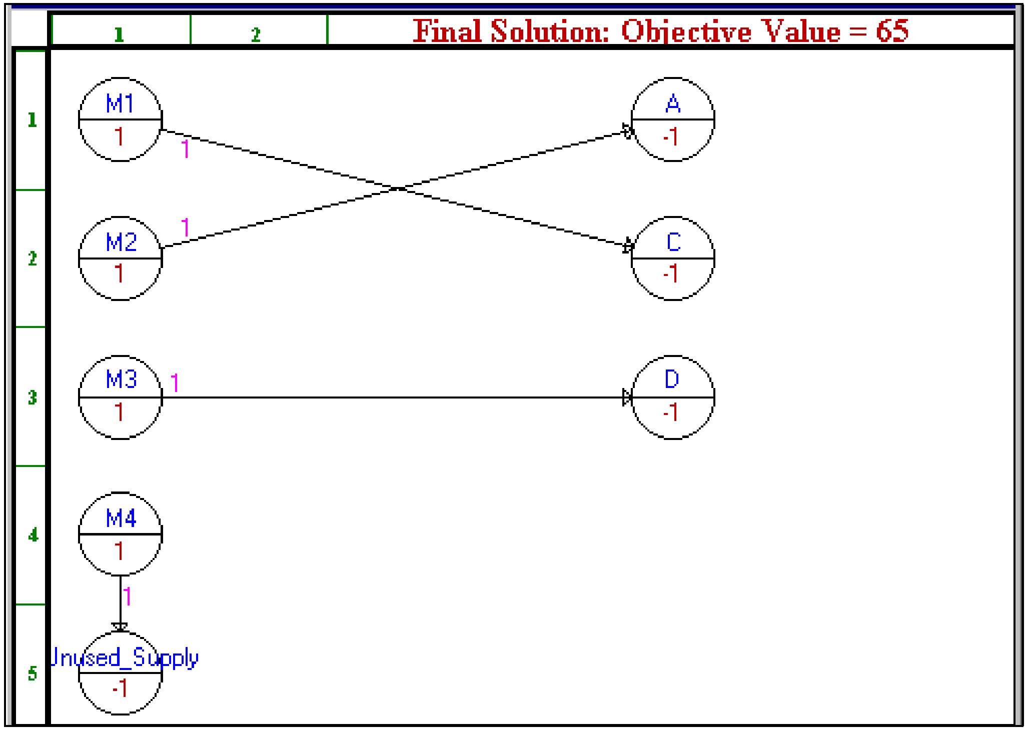


Fig.19. The solution of the problem (case b) in graphic form

distributed. The total distance traveled to the three remaining customers is equal to 65 km. ▣

Problem 3

Four banks B_1, ..., B_4 participate in the financing of four P_1, ..., P_4 projects of a company. Each project is financed by only one bank. The profits of the financing of each project by each bank (in thousands of m.u.) are specified in the Table 3.

Table 3

	P_1	P_2	P_3	P_4
B_1	12	3	18	4
B_2	16	10	14	9
B_3	21	16	25	13
B_4	23	14	21	7

a) Determine an assignment of projects on banks that maximizes the total profit obtained by the company.

b) Analyze how the total profit varies if the profit of the project P_2 financed by the bank B_2 varies in the range [10, 14] thousands of m.u.

a) The above problem is a maximization assignment problem.

The module **Network Modeling** is called for solving. When it completes the **NET Problem Specification** dialog box, it selects the *Maximization option* in the *Objective Criterion* box.

The data of the problem are entered in tabular form in the initial working window (Fig.20).

The command **Solve the Problem** returns the solution of the problem, in tabular form (Fig.21).

From \ To	P1	P2	P3	P4
B1	12	3	18	4
B2	16	10	14	9
B3	21	16	25	13
B4	23	14	21	7

Fig.20. Data of the problem in tabular form

05-06-2013	From	To	Assignment	Unit Profit	Total Profit	Reduced Cost
1	B1	P3	1	18	18	0
2	B2	P4	1	9	9	0
3	B3	P2	1	16	16	0
4	B4	P1	1	23	23	0
	Total	Objective	Function	Value =	66	

Fig.21. The solution of the problem in tabular form

The optimal solution of this assignment problem is the maximum coupling: B_1 - P_3; B_2 - P_4; B_3 - P_2; B_4 - P_1. The maximum value of the total profit of the company is equal to 66 thousands m.u.

b) It calls the command **Perform Parametric Analysis**, which opens the **Parametric Analysis** dialog box, in which the connection B_2 - P_2 is selected and the boxes are filled in with values as in the Fig.22.

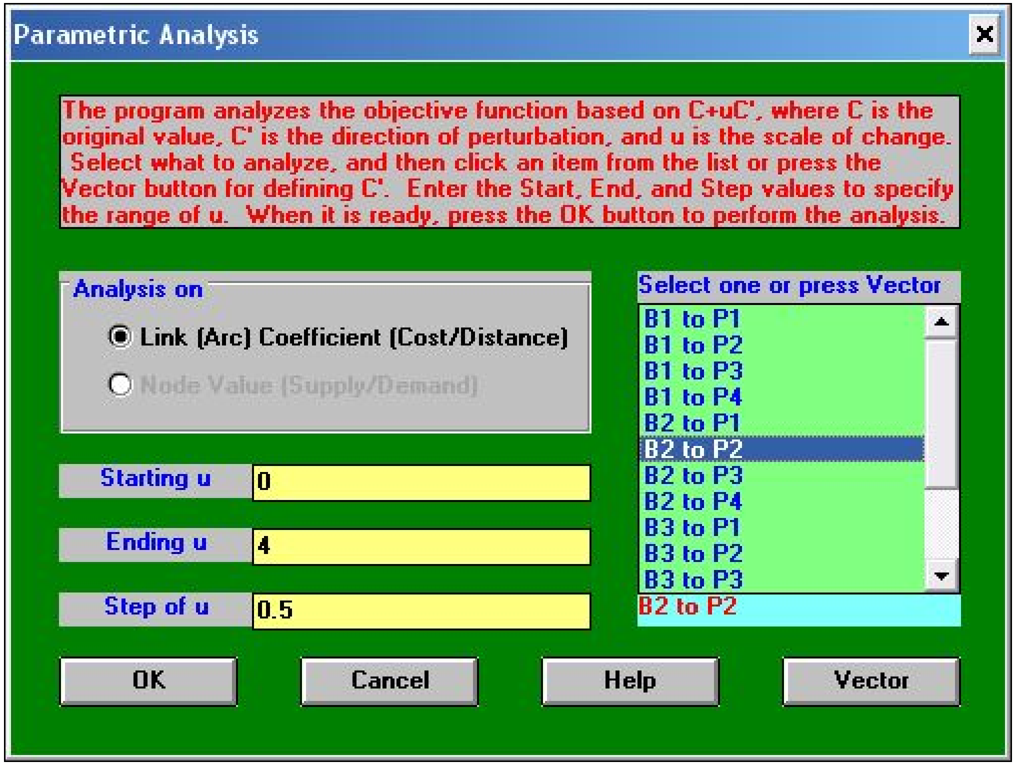


Fig.22. **Parametric Analysis** dialog box

Click on the **OK** command button and it obtains the result of the parametric analysis, in tabular form (Fig.23):

05-06-2013	B2 to P2 Connection Cost/Distance	OBJ Value
1	10	66
2	10.5	66
3	11	66
4	11.5	66
5	12	66
6	12.5	66.5
7	13	67
8	13.5	67.5
9	14	68

Fig.23. The result of the parametric analysis in tabular form

Therefore the total profit varies in the range [66, 68] thousands m.u.

The command **Show Parametric Analysis-Graphic** from the menu **Results** returns the graph of the variation of the total profit value depending on the values of the considered parameter (Fig.24).

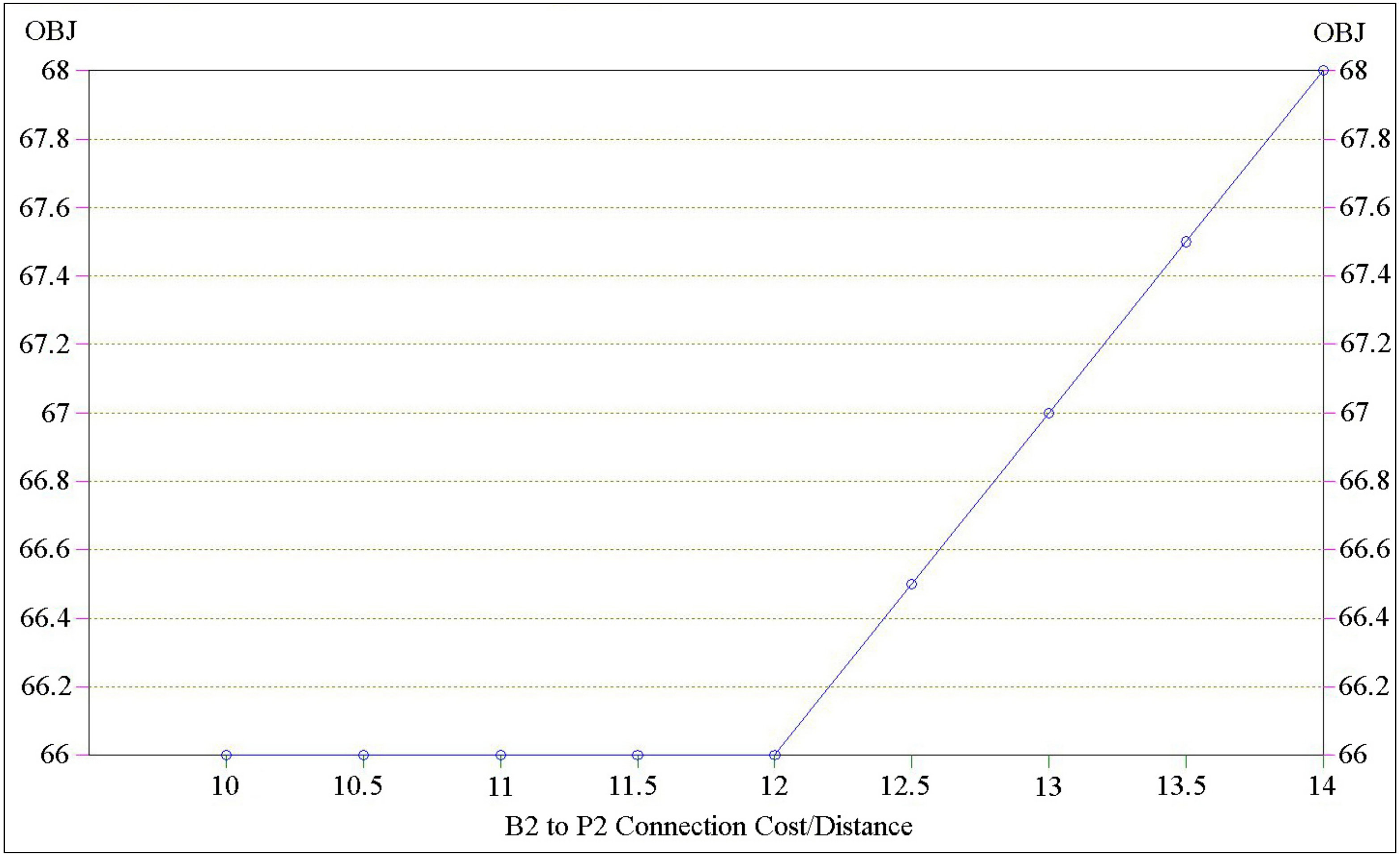


Fig.24. The result of the parametric analysis in graphic form