



University POLITEHNICA of Bucharest

# Applied Mathematics in Optimization Problems

Applications

## Transportation Problem



# **1. Network Modeling module. Transportation Problem type 2. Problems**



# 1. Network Modeling module. Transportation Problem type

The **Network Modeling** module, **Transportation Problem** type, from the product software **WinQSB**, is used to solve, in a conversational system, transportation problems. Its main features are the following:

- It accepts the problem data in tabular form with a predefined structure or in graphical form.
- It displays iteration results in tabular or graphical form.
- It displays the solution of the problem in tabular or graphical form.
- It performs *what ... if ...* analysis and parametric


solution analysis.

- The file with the data problem is saved with the **.net** extension.

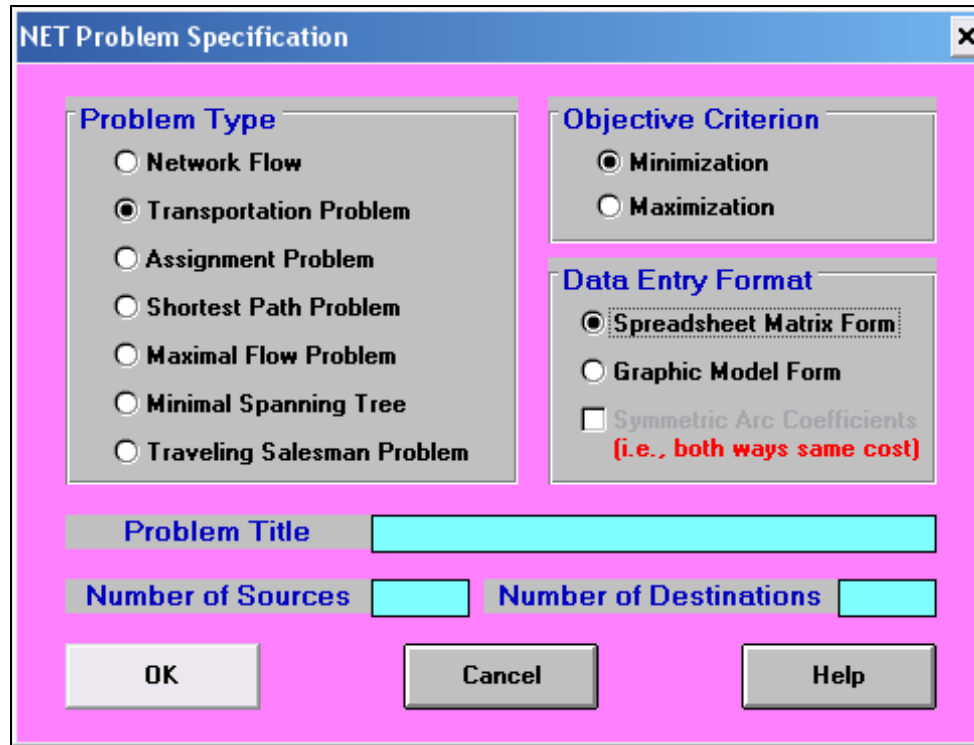
Running the **Network Modeling** module displays the main window of the module:



Fig.1. The main window of the **Network Modeling** module

The **New Problem** command from the **File** menu or a mouse click on the button  displays the **Net Problem**

**Specification** dialog box (Fig.2). It contains fields to be filled in/selected by the user.



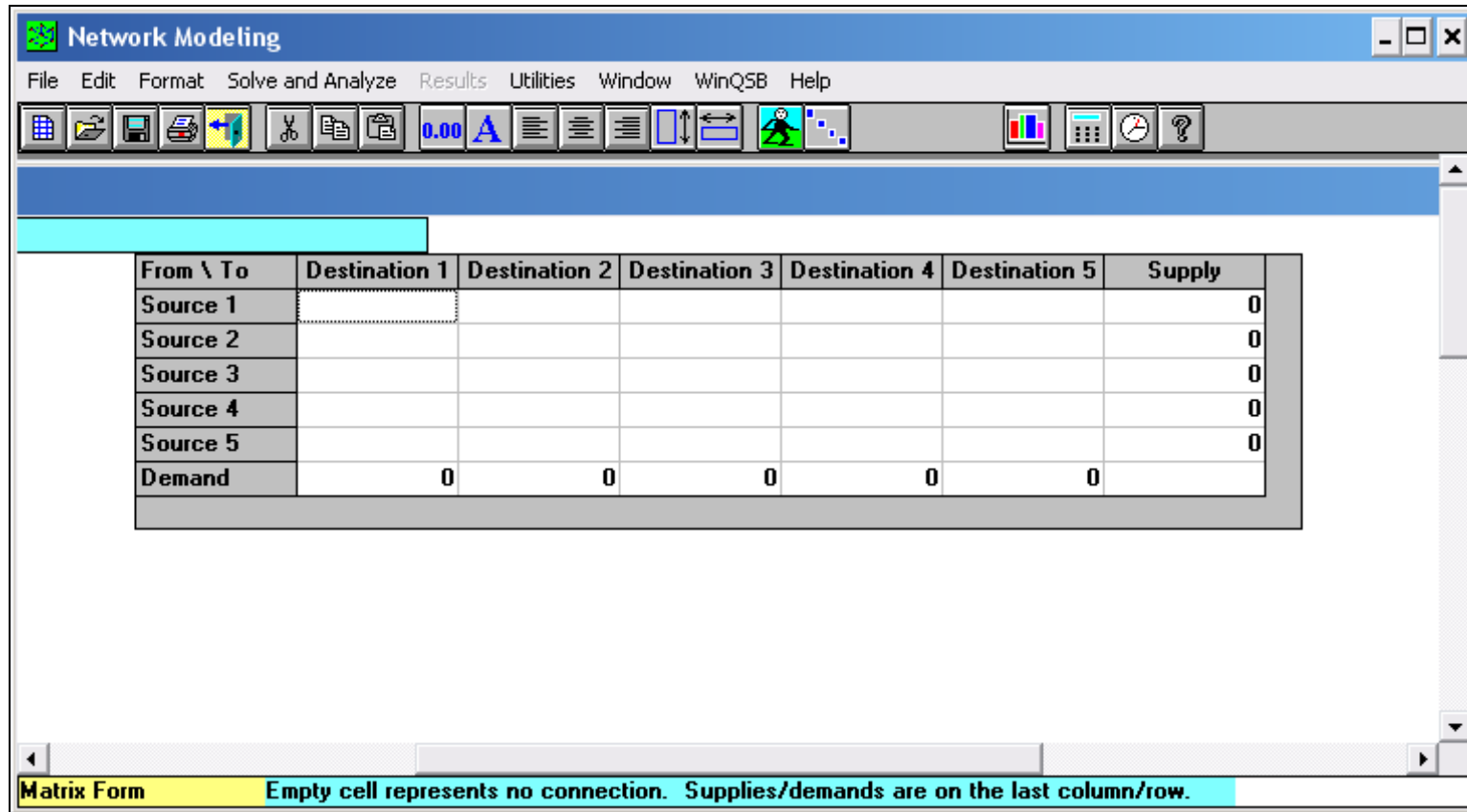
The dialog box, titled "NET Problem Specification", contains the following elements:

- Problem Type:** A list of radio buttons with "Transportation Problem" selected.
  - Network Flow
  - Transportation Problem
  - Assignment Problem
  - Shortest Path Problem
  - Maximal Flow Problem
  - Minimal Spanning Tree
  - Traveling Salesman Problem
- Objective Criterion:** A list of radio buttons with "Minimization" selected.
  - Minimization
  - Maximization
- Data Entry Format:** A list of radio buttons with "Spreadsheet Matrix Form" selected, and a checkbox for "Symmetric Arc Coefficients (i.e., both ways same cost)".
  - Spreadsheet Matrix Form
  - Graphic Model Form
  - Symmetric Arc Coefficients (i.e., both ways same cost)
- Problem Title:** A text input field.
- Number of Sources:** A numeric input field.
- Number of Destinations:** A numeric input field.
- Buttons:** "OK", "Cancel", and "Help".

Fig.2. **Net Problem Specification** dialog box

A click on the command button **OK** in the dialog box

displays the working window, presented in Fig.3:



The screenshot shows the 'Network Modeling' software window. The menu bar includes File, Edit, Format, Solve and Analyze, Results, Utilities, Window, WinQSB, and Help. The toolbar contains various icons for file operations, editing, and solving. The main area displays a transportation problem matrix in 'Matrix Form'. The matrix has 6 rows and 7 columns. The first column is labeled 'From \ To' and the last column is labeled 'Supply'. The rows are labeled 'Source 1' through 'Source 5' and 'Demand'. The columns are labeled 'Destination 1' through 'Destination 5'. The 'Demand' row shows values of 0 for all destinations. The 'Supply' column shows values of 0 for all sources. A status bar at the bottom indicates 'Matrix Form' and provides a note: 'Empty cell represents no connection. Supplies/demands are on the last column/row.'

From \ To	Destination 1	Destination 2	Destination 3	Destination 4	Destination 5	Supply
Source 1						0
Source 2						0
Source 3						0
Source 4						0
Source 5						0
Demand	0	0	0	0	0	

Fig.3. The working window of the module, **Transportation** type

At the top of the working window are:



- *Menu bar*, which contains the menus: **File, Edit, Format, Solve and Analyze, Results, Utilities, Window, WinQSB, Help.**
- *Toolbars* for certain submenus.
- *Bar* with the name of the problem.
- *Bar* indicating the current box.

The menu **Edit** are illustrated in Fig.4.

With the command **Switch to Graphic Model** from the menu **Format**, it switches from the matrix form to the graphic form of a problem.

In the graphic form, the initial working window of a transportation problem is illustrated in the Fig.5. Changing the location of a node can be done by a right-click on

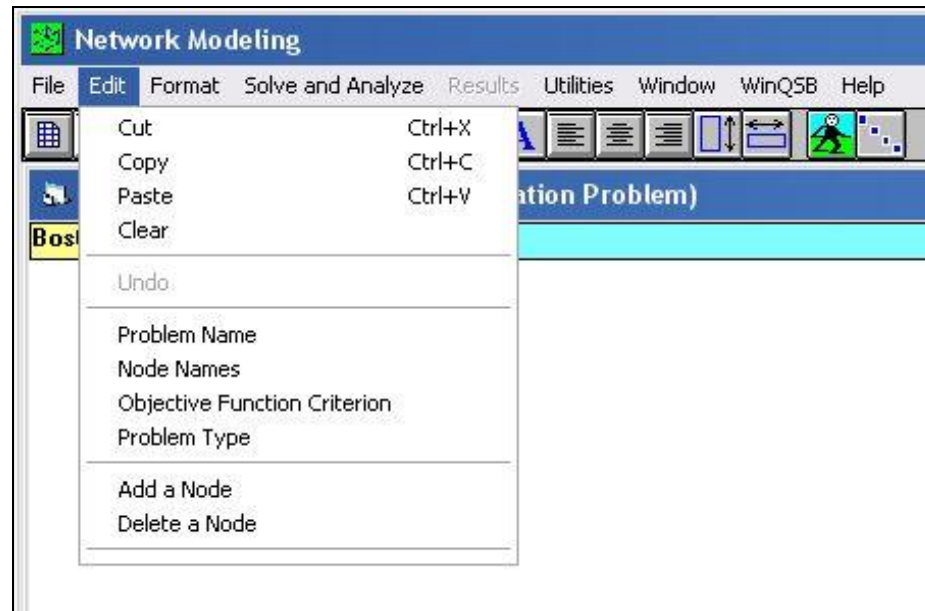


Fig.4. **Edit** menu

the node symbol and pressing the button it moves the node by dragging to the new location.

With the command **Switch to Matrix Form** that appeared in the menu **Format**, it returns from the graphic form to the matrix form.



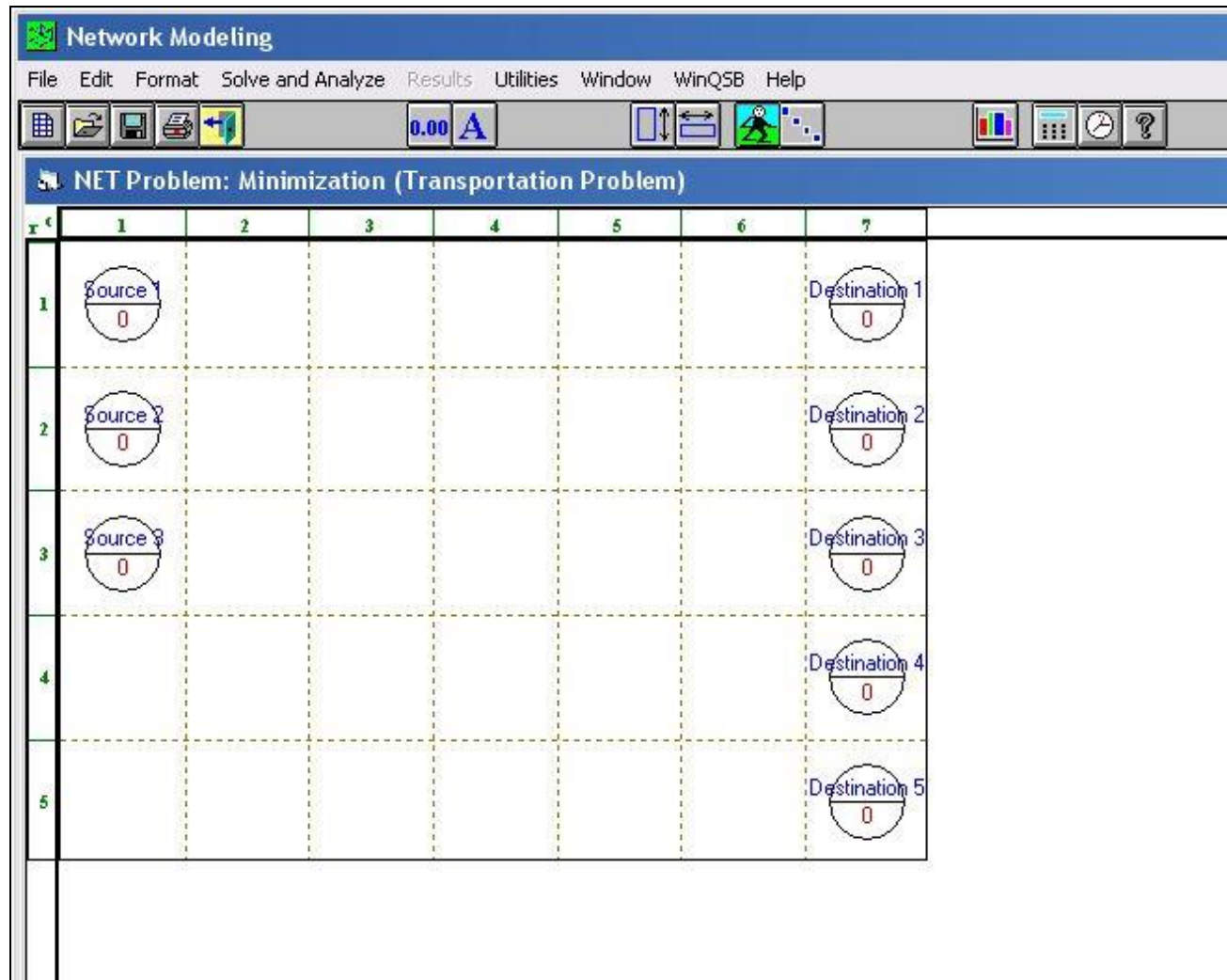


Fig.5. Initial working window in graphic form

The menu **Solve and Analyze** contains commands (Fig.6) related to solving a transportation problem. The command **Select Initial Solution Method** displays a dialog box (Fig.7) with the eight methods for determining an initial solution for problem, that are implemented in the program *WinQSB*.

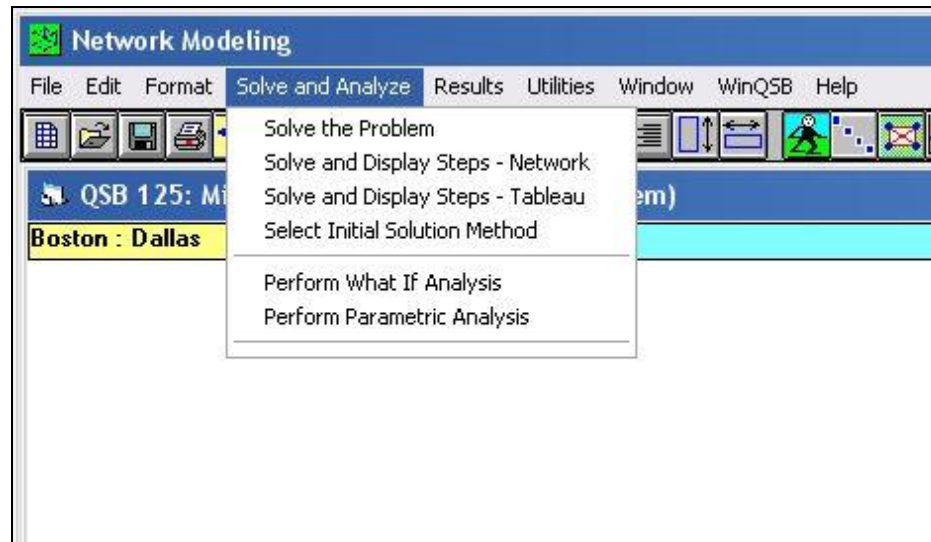


Fig.6. **Solve and Analyze** menu

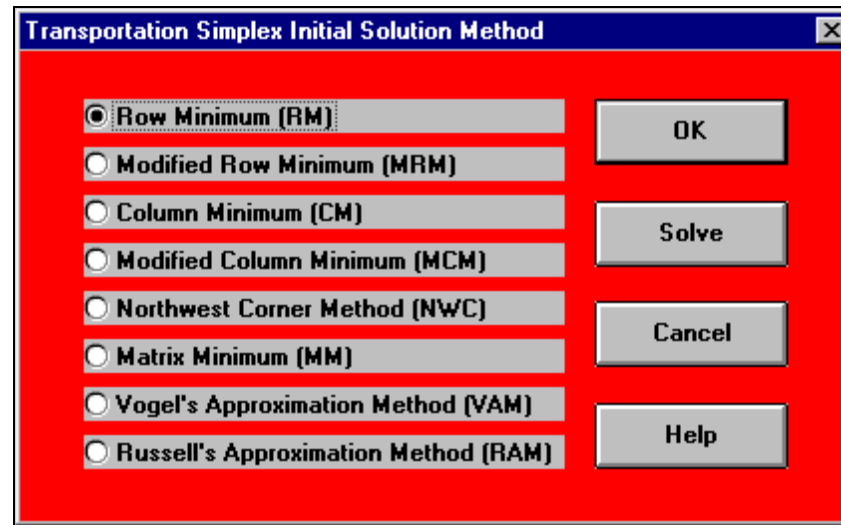


Fig.7. **Transportation Simplex Initial Solution Method** dialog box

The menu **Results**, which is displayed in the results window after solving the problem, contains the commands illustrated in the Fig.8. It should be noted that the optimal solution of the problem is not influenced by the choice of a certain method from those mentioned.



Fig.8. **Results** menu

To solve a transportation problem using the **Network Modeling** module, **Transportation problem** type, the following steps must be completed:

1. Identifying the type of problem.
2. Select the command **New Problem** and complete the




fields in the **NET Problem Specification** dialog box.

3. Entering the data of the problem.
4. Save the data with the command **Save Problem As**.
5. Solve the problem with a command from the menu **Solve and Analyze**.
6. Display the obtained results with a command from the menu **Results**.

## 2. Problems

### Problem 1.

 The warehouses  $D_1, D_2, D_3$  supply the consumers  $C_1, C_2, C_3, C_4$  with a certain product. The quantities available in warehouses  $a_i$  ( $i = 1, \dots, 3$ ), the quantities needed by consumers  $b_j$  ( $j = 1, \dots, 4$ ) and the unit transportation costs of the product  $c_{ij}$  are presented in the Table 1.

a) Determine the quantities to be transported from each warehouse to each consumer so that the total cost of transporting is minimal.

b) Perform the parametric analysis for the following cases:  $a_2 \in [20, 95]$ ;  $b_4 \in [25, 90]$ ;  $c_{31} \in [1, 10]$ .

Table 1

Warehouses	Consumers				Supply
	$C_1$	$C_2$	$C_3$	$C_4$	
$D_1$	6	4	3	5	100
$D_2$	3	7	6	7	70
$D_3$	4	3	4	5	80
Demand	75	40	75	60	-



a) This minimization transportation problem is balanced because:

$$\sum_{i=1}^3 a_i = \sum_{j=1}^4 b_j = 250 \text{ u.}$$

The **Network Modeling** module is called for its solving. It selects the command **New Problem** and it

completes the **NET Problem Specification** dialog box as in the Fig.9.

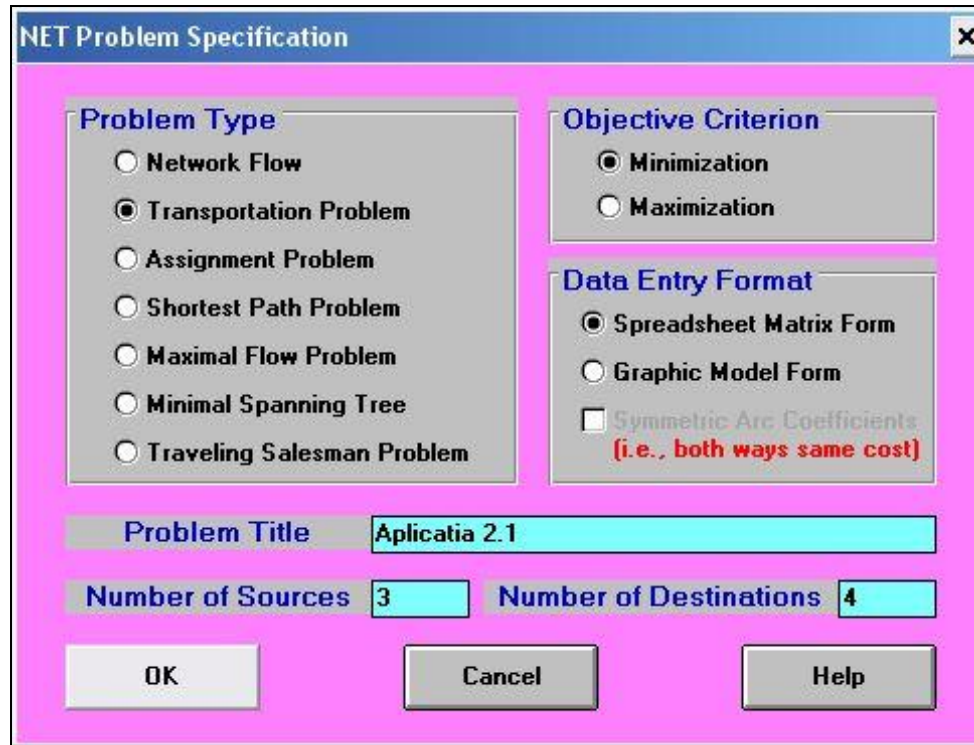


Fig.9. **NET Problem Specification** dialog box

Then click the command button **OK**. The data of the



problem in tabular form is presented in the Fig.10.

From \ To	C1	C2	C3	C4	Supply
D1	6	4	3	5	100
D2	3	7	6	7	70
D3	4	3	4	5	80
Demand	75	40	75	60	

Fig.10. The data of the problem in tabular form

The table with the optimal solution of the problem, obtained in the results window with the command **Solve the Problem** after a single iteration, is presented in the Fig.11. The column *Shipment* contains the optimal values of the (basic) variables, i.e. the quantities of product to be transported from warehouses to consumers. The transportation costs corresponding for these quantities are indicated in the column *Total Cost*.



09-30-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C3	75	3	225	0
2	D1	C4	25	5	125	0
3	D2	C1	70	3	210	0
4	D3	C1	5	4	20	0
5	D3	C2	40	3	120	0
6	D3	C4	35	5	175	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>875</b>	

Fig.11. Optimal solution of the problem in tabular form

The minimum value of the total cost of transporting is equal to 875 m.u.

The command **Range of Optimality** from the menu **Results** returns the table in the Fig.12. The columns *Allowable Min. Cost* and *Allowable Max. Cost* contain the minimum allowable value, respectively, the maximum allowable value of the unit transportation costs, for which the above optimal solution do not change.



12-14-2011 11:45:21	From	To	Unit Cost	Reduced Cost	Basis Status	Allowable Min. Cost	Allowable Max. Cost
1	D1	C1	6	2	at bound	4	M
2	D1	C2	4	1	at bound	3	M
3	D1	C3	3	0	basic	0	4
4	D1	C4	5	0	basic	5	6
5	D2	C1	3	0	basic	-M	4
6	D2	C2	7	5	at bound	2	M
7	D2	C3	6	4	at bound	2	M
8	D2	C4	7	3	at bound	4	M
9	D3	C1	4	0	basic	3	6
10	D3	C2	3	0	basic	0	4
11	D3	C3	4	1	at bound	3	M
12	D3	C4	5	0	basic	4	5

Fig.12. Table returned by the command **Range of Optimality**

The command **Range of Feasibility** returns the table in Fig.13. The values of the variables of the dual problem are given in the column *Shadow Price*. The columns *Allowable Min. Value* and *Allowable Max. Value* contains the minimum possible value, respectively, the maximum possible value for the available / required quantities of the



12-14-2011 11:54:55	Node	Supply	Demand	Shadow Price	Allowable Min. Value	Allowable Max. Value
1	D1	100	0	0	100	M
2	D2	70	0	-1	70	75
3	D3	80	0	0	80	105
4	C1	0	75	4	70	75
5	C2	0	40	3	15	40
6	C3	0	75	3	0	75
7	C4	0	60	5	35	60

Fig.13. Table returned by the command **Range of Feasibility**

product. If a quantity  $a_i$  or  $b_j$  takes another value in that range, different from the one in the problem statement, then the optimal value of the total shipping cost (875) changes with the product result between the value in the column *Shadow Price* and the difference between the new value and the one in the statement of the quantity  $a_i$  or  $b_j$ . Obviously, if the corresponding shadow price is 0, then the value of the total shipping cost does not change.



b) To perform the parametric analysis, after solving the problem, it calls the command **Perform Parametric Analysis** from the menu **Results**, which displays the **Parametric Analysis** dialog box.

For  $a_2 \in [20, 95]$ , it fills the **Parametric Analysis** dialog box, in the way illustrated in the Fig.14. A click on the **OK** button returns the result of the parametric analysis in tabular form (see Fig.15). The graphic image of the parametric analysis (see Fig.16) is obtained with the command **Show Parametric Analysis - Graphic**.

For  $b_4 \in [25, 90]$ , it fills the **Parametric Analysis** dialog box, in the way illustrated in the Fig.17. The result of the parametric analysis in graphic format is presented

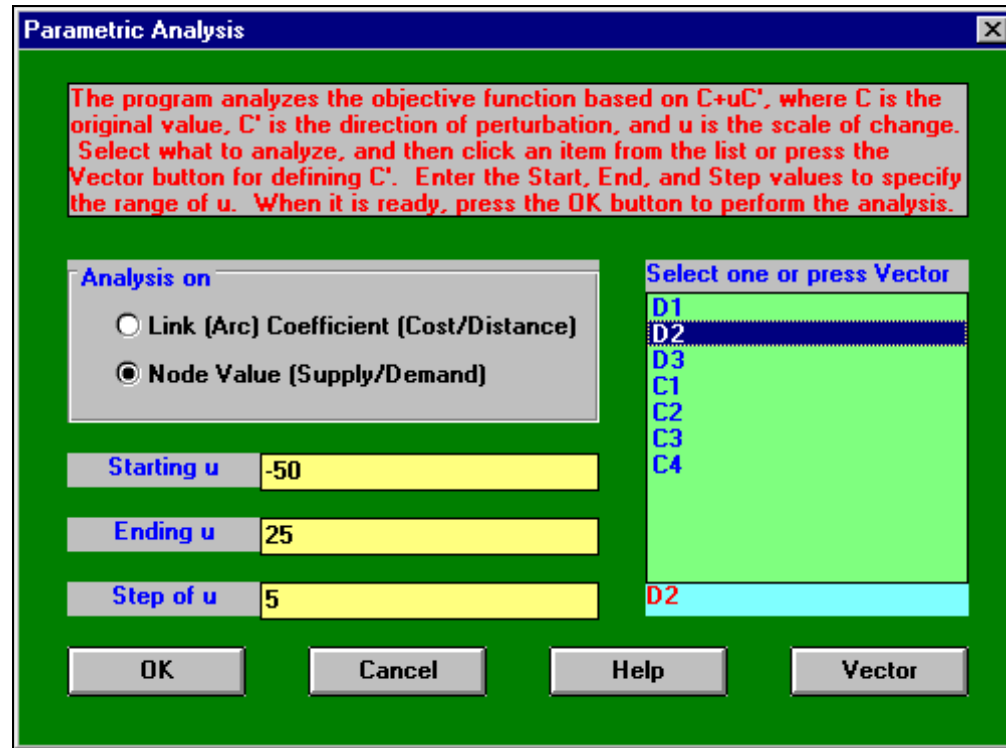


Fig.14. **Parametric Analysis** dialog box  
for  $a_2 \in [20, 95]$

12-15-2011	D2 Supply/Demand	OBJ Value
1	20	690
2	25	705
3	30	720
4	35	735
5	40	755
6	45	775
7	50	795
8	55	815
9	60	835
10	65	855
11	70	875
12	75	870
13	80	870
14	85	870
15	90	870
16	95	870

Fig.15. The result of the parametric analysis, in tabular form, for  $a_2 \in [20, 95]$

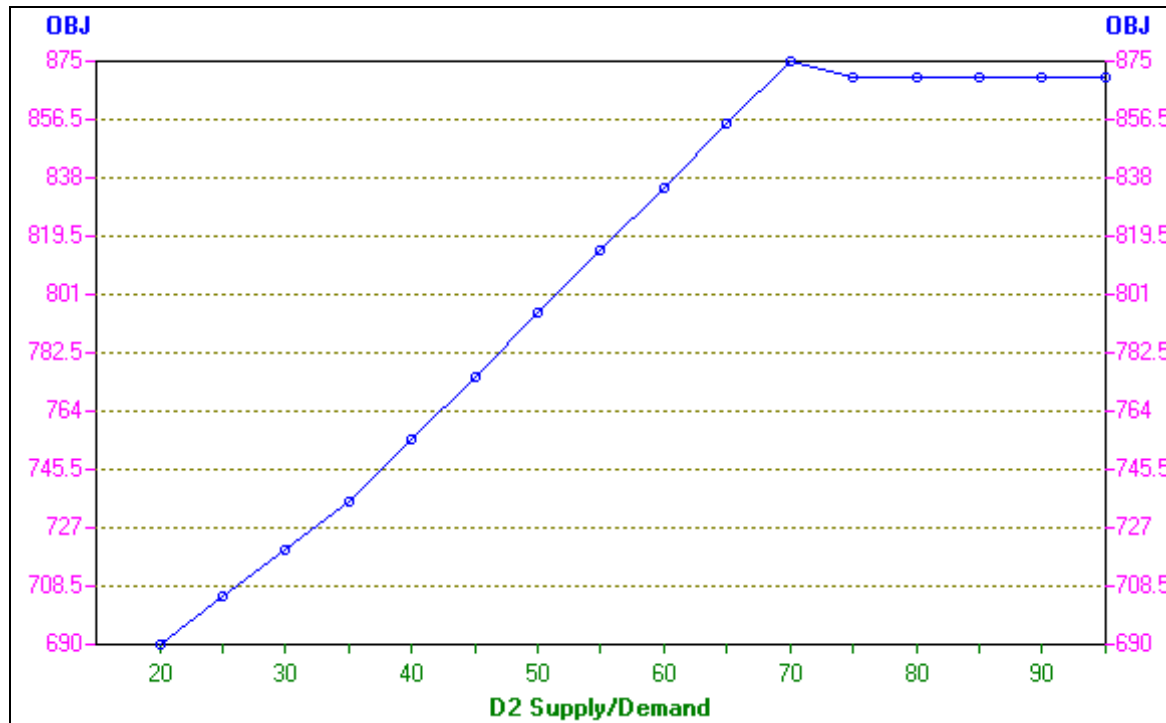


Fig.16. The result of the parametric analysis, in graphic form, for  $a_2 \in [20, 95]$



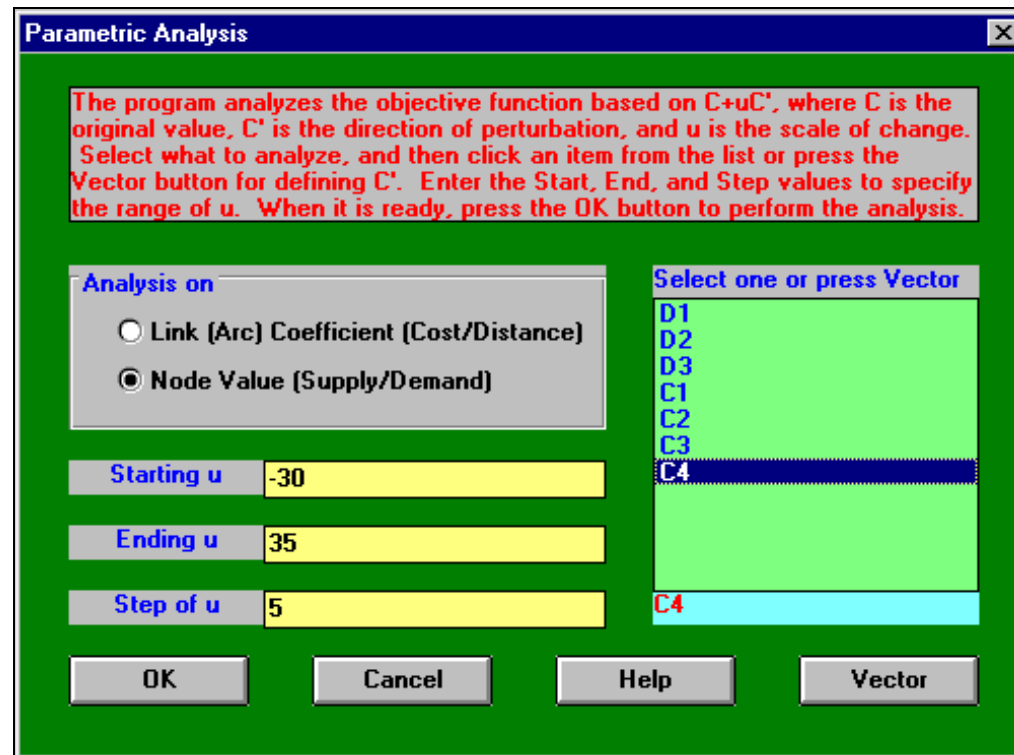


Fig.17. **Parametric Analysis** dialog box  
for  $b_4 \in [25, 90]$

in the Fig.18:

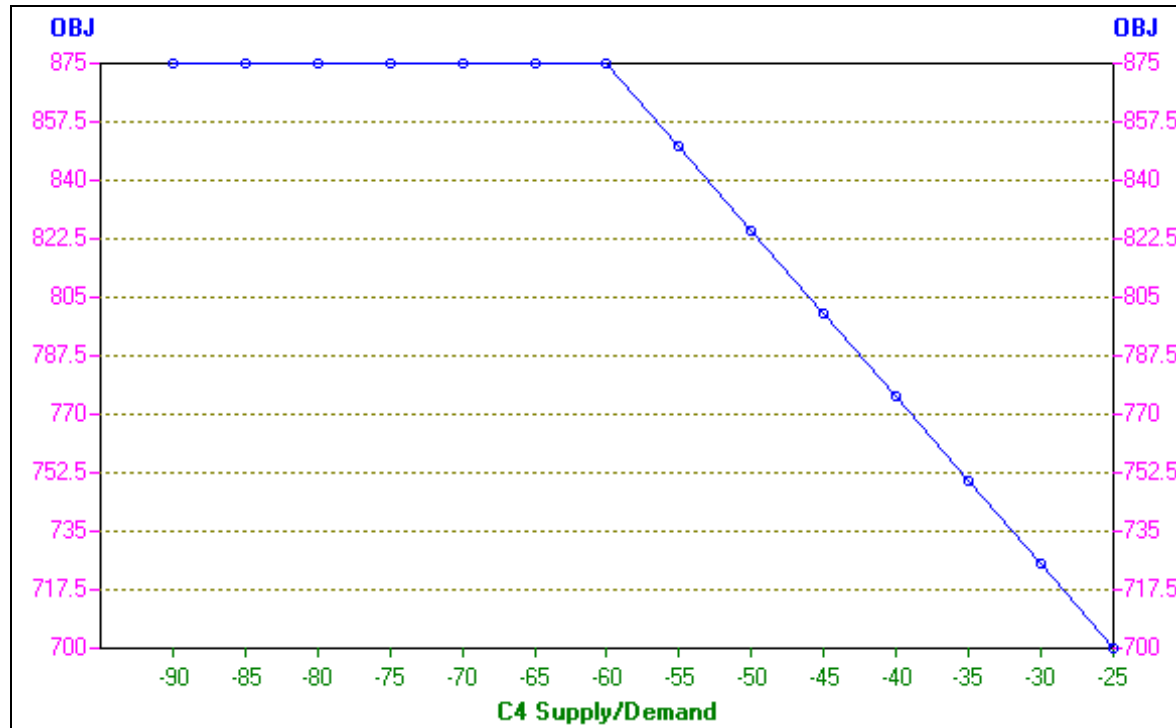


Fig.18. The result of the parametric analysis,  
in graphic form, for  $b_4 \in [25, 90]$

For  $c_{31} \in [1, 10]$ , it fills the **Parametric Analysis** dialog box, in the way illustrated in the Fig.19:

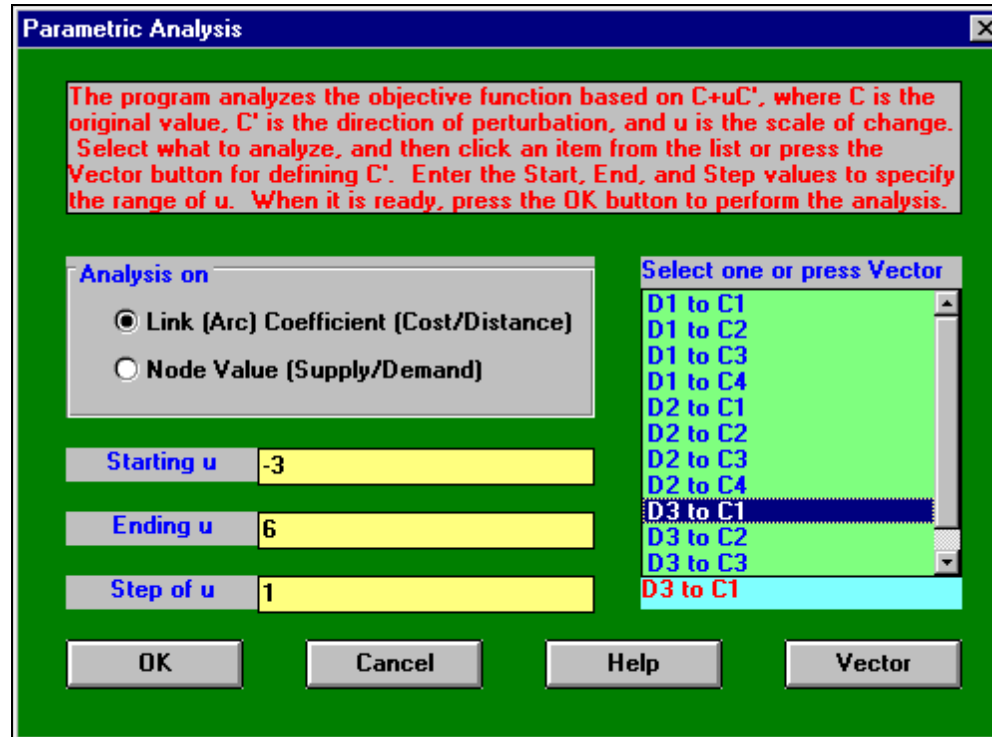


Fig.19. **Parametric Analysis** dialog box for  $c_{31} \in [1, 10]$

The result of the parametric analysis in graphic format is presented in the Fig.20: 

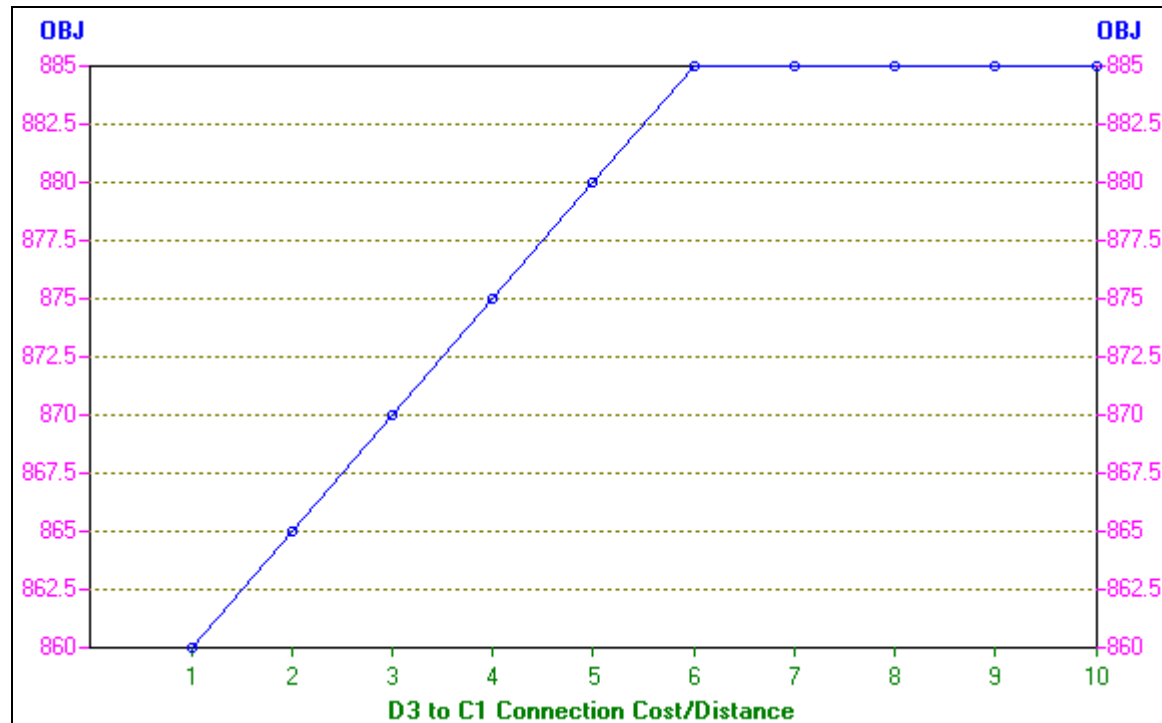


Fig.20. The result of the parametric analysis, in graphic form, for  $c_{31} \in [1, 10]$

## Problem 2


 Solving the minimum total cost transportation problem whose data are summarized in the Table 3.

Table 3

Warehouses	Consumers				Supply
	$C_1$	$C_2$	$C_3$	$C_4$	
$D_1$	6	3	2	5	150
$D_2$	7	8	7	9	200
$D_3$	4	3	2	5	180
$D_4$	5	6	3	7	120
Demand	160	140	190	160	-



This minimization transportation problem is balanced.

The module **Network Modeling** is called for solving

the problem.

After completing the **NET Problem Specification** dialog box, the data are entered in tabular form in the initial window:

From \ To	C1	C2	C3	C4	Supply
D1	6	3	2	5	150
D2	7	8	7	9	200
D3	4	3	2	5	180
D4	5	6	3	7	120
Demand	160	140	190	160	

Fig.21. The data of the problem in tabular form

Proceeding as in the Problem 1, the command **Solve the Problem** returns the optimal solution, in tabular form (Fig.22):  $x^1 = (x_{12} = 80 \text{ u.}, x_{13} = 70 \text{ u.}, x_{21} = 160 \text{ u.}, x_{24} = 40 \text{ u.}, x_{32} = 60 \text{ u.}, x_{34} = 120 \text{ u.}, x_{43} = 120 \text{ u.})$ . The total cost of transportation is equal to 3000 m.u.



12-15-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C2	80	3	240	0
2	D1	C3	70	2	140	0
3	D2	C1	160	7	1120	0
4	D2	C4	40	9	360	0
5	D3	C2	60	3	180	0
6	D3	C4	120	5	600	0
7	D4	C3	120	3	360	0
	Total	Objective	Function	Value =	3000	

Fig.22. The optimal solution  $x^1$  in tabular form

The problem supports multiple solutions because, after obtaining the above solution, the command **Obtain Alternative Solution** appears in the menu **Solve and Analyze**. This command returns another optimal solution, in tabular form (Fig.23):  $x^2 = (x_{13} = 70 \text{ u.}, x_{14} = 80 \text{ u.}, x_{21} = 160 \text{ u.}, x_{24} = 40 \text{ u.}, x_{32} = 140 \text{ u.}, x_{34} = 40 \text{ u.}, x_{43} = 120 \text{ u.})$ . Obvious, the total cost of transportation is equal to 3000 m.u.



12-15-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C3	70	2	140	0
2	D1	C4	80	5	400	0
3	D2	C1	160	7	1120	0
4	D2	C4	40	9	360	0
5	D3	C2	140	3	420	0
6	D3	C4	40	5	200	0
7	D4	C3	120	3	360	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>3000</b>	

Fig.23. The optimal solution  $x^2$  in tabular form


The set of the optimal solutions is given by the convex linear combination:

$$x = \lambda x^1 + (1 - \lambda)x^2, \quad \lambda \in [0, 1]$$

The total transportation cost is equal to 3000 m.u. 



## Problem 3


 A company specialized in commercialization a certain product on the market has three warehouses  $D_i$  ( $i = 1, \dots, 3$ ) and four stores  $M_j$  ( $j = 1, \dots, 4$ ). The warehouses have 250 pcs., 400 pcs., respectively, 300 pcs. of the product. The stores can sell at most 230 pcs., 200 pcs., 240 pcs., respectively, 230 pcs. of the product.

The unit profit obtained by the company by selling the product depends on the warehouse from which it is brought and on the store that sells it, according to the Table 4.

Determine the optimal solution that maximizes the total profit of the company resulting from the sale of the product.

Table 4

	$M_1$	$M_2$	$M_3$	$M_4$
$D_1$	16	14	11	13
$D_2$	12	15	16	14
$D_3$	13	12	14	15

 The statement problem is of transport type, of maximization, unbalanced, with an surplus offer, having  $950 - 900 = 50$  pcs. of surplus product in warehouses supply compared to stores demand.

The **Network Modeling** module is called for solving.

When it completes the **NET Problem Specification** dialog box, in the box *Objective Criterion*, it selects the *Maximization* option.

In the initial work window, the data of the are entered in tabular form, as seen in the Fig.24:

From \ To	M1	M2	M3	M4	Supply
D1	16	14	11	13	250
D2	12	15	16	14	400
D3	13	12	14	15	300
Demand	230	200	240	230	

Fig.24. The data of the problem in tabular form

Switching to the graphic form (see Fig.25) is performed with the command **Switch to Graphic Model** from the menu **Format**.

The optimal solution of the problem is obtained, in tabular form (see Fig.26) with the command **Solve the Problem**.

The command **Graphic Solution** from the menu

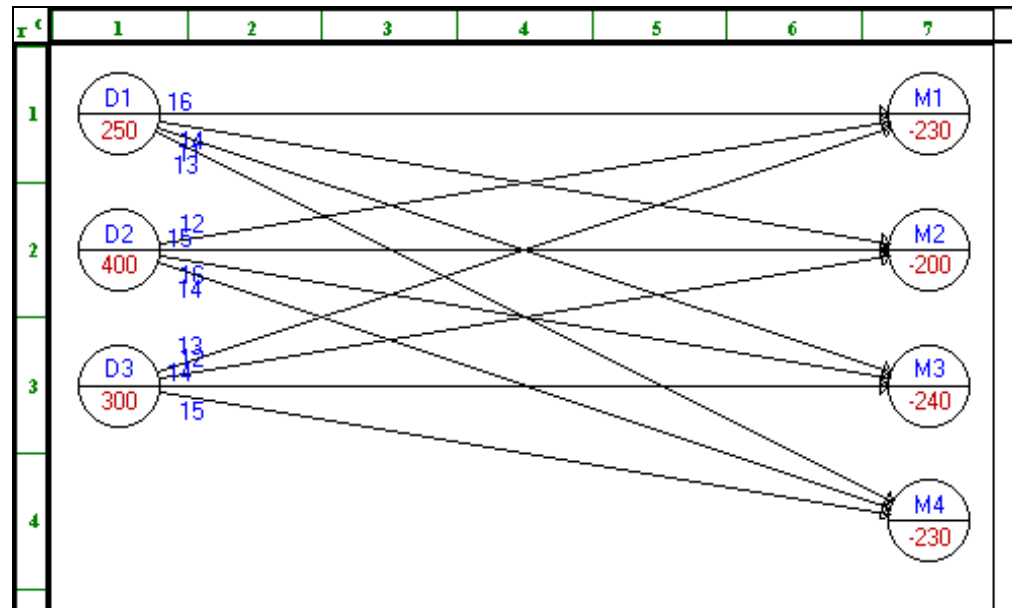


Fig.25. The data of the problem in graphic form

12-16-2011	From	To	Shipment	Unit Profit	Total Profit	Reduced Cost
1	D1	M1	230	16	3680	0
2	D1	M2	20	14	280	0
3	D2	M2	180	15	2700	0
4	D2	M3	220	16	3520	0
5	D3	M3	20	14	280	0
6	D3	M4	230	15	3450	0
7	D3	Unused_Supply	50	0	0	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>13910</b>	

Fig.26. The optimal solution in tabular form

**Results** transfers the solution of the problem from the tabular form to the graphic form (Fig.27). The surplus of 50 pcs. of product in storage is concentrated in the fictive node, called "*Unused\_Supply*".

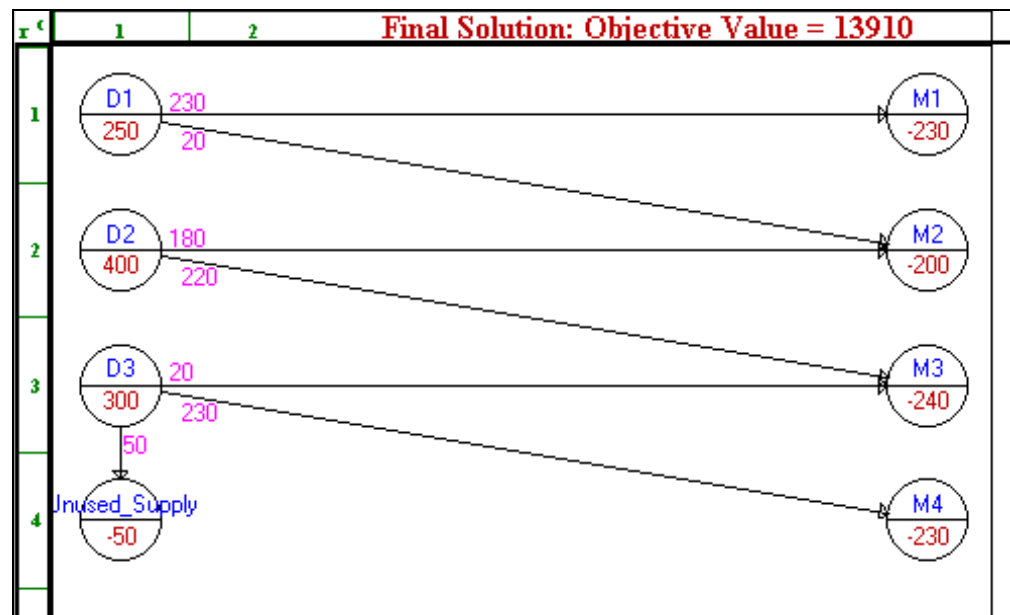


Fig.27. The optimal solution in graphic form

## Problem 4


 The suppliers  $D_i$  ( $i = 1, \dots, 3$ ) have a certain product requested by the beneficiaries  $C_j$  ( $j = 1, \dots, 5$ ). The available quantities, the necessary quantities and the unit cost of transportation from a supplier to a beneficiary are given in the Table 5.

Table 5

	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	Supply
$D_1$	65	55	40	35	50	340
$D_2$	40	25	35	50	25	300
$D_3$	50	40	35	45	45	260
Demand	160	190	200	170	180	-

a) Determine the optimal product supply plan so that the total cost of transportation is minimal.

- b) Establish the supply plan if the unit transport cost  $c_{31}$  becomes equal to 60 m.u.
- c) Solve the problem if the route  $(D_2, C_2)$  cannot be used for the transport of the product.
- d) Solve the problem provided that the quantity  $x_{34} = 120$  u. is transported on the route  $(D_3, C_4)$ .



- a) The statement problem is of minimization and unbalanced.

The data of the problem are entered in the tabular form

From \ To	C1	C2	C3	C4	C5	Supply
D1	65	55	40	35	50	340
D2	40	25	35	50	25	300
D3	50	40	35	45	45	260
Demand	160	190	200	170	180	

Fig.28. The data of the problem in tabular form

The optimal solution of the problem in tabular form is presented in Fig.30.

10-16-2020	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C3	170	40	6800	0
2	D1	C4	170	35	5950	0
3	D2	C2	120	25	3000	0
4	D2	C5	180	25	4500	0
5	D3	C1	160	50	8000	0
6	D3	C2	70	40	2800	0
7	D3	C3	30	35	1050	0
	Total	Objective	Function	Value =	32100	

Fig.30. The optimal solution of the problem in tabular form

b) The command **Perform What If Analysis** from the menu **Results** opens the **What If Analysis** dialog box, which is completed as in the Fig.31. A click on the **OK** button displays the new solution (Fig.32). The total transportation cost increases from 32100 m.u. at 33100 m.u.



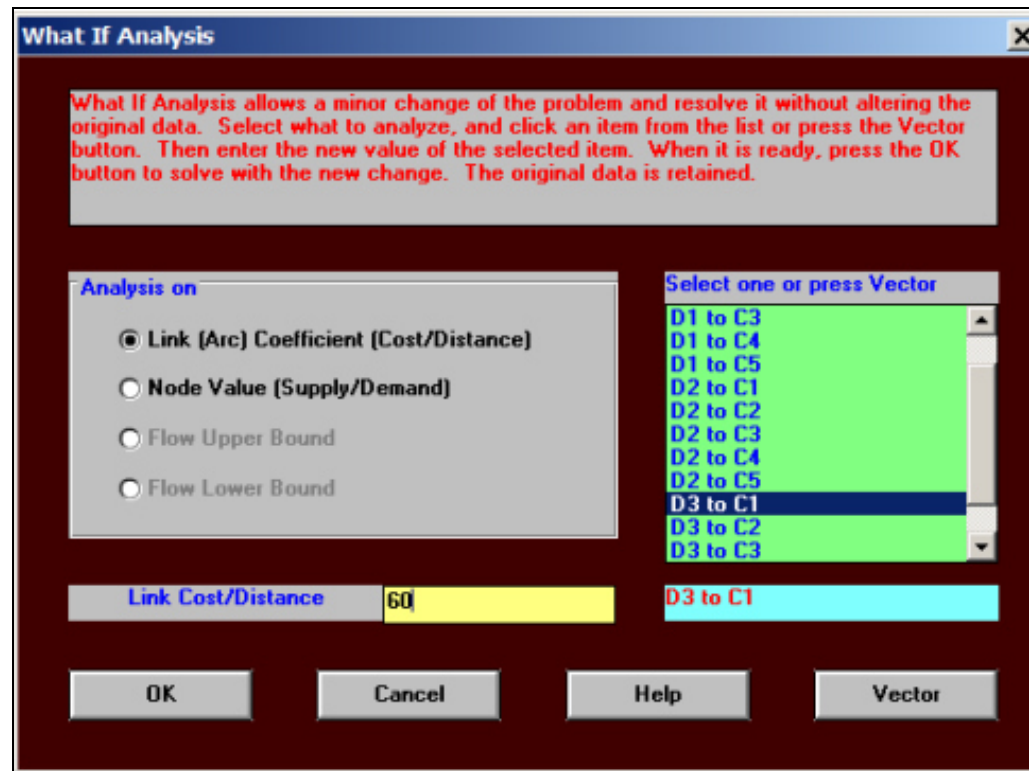


Fig.31. What If Analysis dialog box



10-16-2020	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C3	170	40	6800	0
2	D1	C4	170	35	5950	0
3	D2	C1	120	40	4800	0
4	D2	C5	180	25	4500	0
5	D3	C1	40	60	2400	0
6	D3	C2	190	40	7600	0
7	D3	C3	30	35	1050	0
	Total	Objective	Function	Value =	33100	

Fig.32. The optimal solution of the problem in case b)

c) In this case the transportation problem has a blocked route. In order for that route not to be used, a very high unit cost,  $M \gg 0$ , is assigned to the route  $(D_2, C_2)$  in the table with the input data (see the Fig.33).

The command **Solve the Problem** returns the optimal solution in tabular form (Fig.34).



From \ To	C1	C2	C3	C4	C5	Supply
D1	65	55	40	35	50	340
D2	40	M	35	50	25	300
D3	50	40	35	45	45	260
Demand	160	190	200	170	180	

Fig.33. The data of the problem with the route  $(D_2, C_2)$  blocked

10-16-2020	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C3	170	40	6800	0
2	D1	C4	170	35	5950	0
3	D2	C1	120	40	4800	0
4	D2	C5	180	25	4500	0
5	D3	C1	40	50	2000	0
6	D3	C2	190	40	7600	0
7	D3	C3	30	35	1050	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>32700</b>	

Fig.34. The optimal solution of the problem in case c)



d) In this case we have a transport problem with imposed solution. For the solution it is considered that the transport of the 120 u. on the route  $(D_3, C_4)$  was done in advance. Then:

- the supplier  $D_3$  has the quantity:  $260 - 120 = 140$  u.;
- the beneficiary  $C_4$  requests the quantity:  $170 - 120 = 50$  u .;
- the route  $(D_3, C_4)$  is blocked, by the unit cost  $c_{34} = M$ , with  $M \gg 0$ .

The transportation problem thus modified is solved. The data of the problem are entered in tabular form (Fig.35). After solving, the obtained solution is presented in the Fig.36.



From \ To	C1	C2	C3	C4	C5	Supply
D1	65	55	40	35	50	340
D2	40	25	35	50	25	300
D3	50	40	35	M	45	140
Demand	160	190	200	50	180	

Fig.35. The data of the modified problem

10-16-2020	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	D1	C3	200	40	8000	0
2	D1	C4	50	35	1750	0
3	D1	C5	90	50	4500	0
4	D2	C1	20	40	800	0
5	D2	C2	190	25	4750	0
6	D2	C5	90	25	2250	0
7	D3	C1	140	50	7000	0
	Total	Objective	Function	Value =	29050	

Fig.36. The optimal solution of the problem in case d)

The optimal solution of the problem with imposed solution is given by the previous solution to which is added the imposed component  $x_{34} = 120$  u.



Therefore the solution is:

$$x_{13} = 200 \text{ u.}, x_{14} = 50 \text{ u.}, x_{15} = 90 \text{ u.}, x_{21} = 20 \text{ u.}, \\ x_{22} = 190 \text{ u.}, x_{25} = 90 \text{ u.}, x_{31} = 140 \text{ u.}, x_{34} = 120 \text{ u.}$$


The total transportation cost is:

$$z = 29050 + 45 \times 120 = 34450 \text{ m.u.}$$





## Problem 5

 A company that produces refrigerators has two factories,  $F_1$  and  $F_2$ , located in two localities. During a quarter, the factory  $F_1$  can produce up to 400 pcs, and the factory  $F_2$  can produce up to 500 pcs.

The refrigerators are sent to two warehouses  $D_1$  and  $D_2$ , located in two other localities. The unit transportation costs from the two factories to the warehouses are indicated in the Table 6.

Four sales centers  $C_1, \dots, C_4$ , located in different localities, request in a quarter 230 pcs, 200 pcs, 220 pcs, respectively, 250 pcs of refrigerators. These are sent from the warehouses to the sales centers, with the unit costs indicated in the Table 7.

There is the possibility that some centers will also receive the refrigerators directly from the two factories, with unit transportation costs indicated in the Table 8.

What is the transportation plan for the refrigerators from the factories to the sales centers with the minimum total cost ?

Table 6

	$D_1$	$D_2$
$F_1$	3	4
$F_2$	3	2


Table 7

	$C_1$	$C_2$	$C_3$	$C_4$
$D_1$	6	4	2	7
$D_2$	5	3	3	6

Table 8

	$C_1$	$C_2$	$C_3$	$C_4$
$F_1$	-	7	-	9
$F_2$	7	-	8	-



 The statement problem is a transfer problem (transportation problem with intermediate or transit centers).

The available quantity of refrigerators is equal to the required quantity  $Q = 900$  pcs., so the problem is balanced.

The transfer problem is an extension of the transportation problem. It can be solved on an extended table of the transportation problem; in this case the extended table is presented in the Fig.37.

The intermediate (transit) centers are the warehouses  $D_1$  and  $D_2$ . Therefore, their available and necessary quantities are equal to  $Q = 900$  pcs.

From \ To	D 1	D 2	C 1	C 2	C 3	C 4	Supply
F 1	3	4	M	7	M	9	400
F 2	3	2	7	M	8	M	500
D 1	0	M	6	4	2	7	900
D 2	M	0	5	3	3	6	900
Demand	900	900	230	200	220	250	

Fig.38. The extend tabel of the transportation problem

The unit cost of transportation from a warehouse to itself is zero.

The unit costs equal to  $M \gg 0$  block those routes from factories to consumption centers, which, according to the data in the statement, cannot be used.

Two optimal solutions, presented in the Fig.38 and Fig.39, are obtained.

The routes  $(i, i)$  are not considered.



-23-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	F 1	D 1	220	3	660	0
2	F 1	C 4	180	9	1620	0
3	F 2	D 2	500	2	1000	0
4	D 1	D 1	680	0	0	0
5	D 1	C 3	220	2	440	0
6	D 2	D 2	400	0	0	0
7	D 2	C 1	230	5	1150	0
8	D 2	C 2	200	3	600	0
9	D 2	C 4	70	6	420	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>5890</b>	

Fig.38. The optimal first solution of the extended problem

-23-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	F 1	D 1	220	3	660	0
2	F 1	C 4	180	9	1620	0
3	F 2	D 2	270	2	540	0
4	F 2	C 1	230	7	1610	0
5	D 1	D 1	680	0	0	0
6	D 1	C 3	220	2	440	0
7	D 2	D 2	630	0	0	0
8	D 2	C 2	200	3	600	0
9	D 2	C 4	70	6	420	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>5890</b>	

Fig.39. The optimal second solution of the extended problem



The first optimal solution is:

- 220 pcs. are sent from  $F_1$  to  $D_1$  ;
- 180 pcs. are directly sent from  $F_1$  to  $C_4$  ;
- 500 pcs. are sent from  $F_2$  to  $D_2$  ;
- 220 pcs. are sent from  $D_1$  to  $C_3$  ;
- 230 pcs. are sent from  $D_2$  to  $C_1$  ;
- 200 pcs. are sent from  $D_2$  to  $C_2$  ;
- 70 pcs. are sent from  $D_2$  to  $C_4$ .

The second optimal solution is:


- 220 pcs. are sent from  $F_1$  to  $D_1$  ;
- 180 pcs. are directly sent from  $F_1$  to  $C_4$  ;
- 270 pcs. are sent from  $F_2$  to  $D_2$  ;
- 230 pcs. are directly sent from  $F_2$  to  $C_1$  ;
- 220 pcs. are sent from  $D_1$  to  $C_3$  ;



200 pcs. are sent from  $D_2$  to  $C_2$  ;  
70 pcs. are sent from  $D_2$  to  $C_4$  .  
The minimum total cost is equal to 5890 m.u.



## Problem 6

 Six pumping stations  $P_1, \dots, P_6$  in a water supply network are connected to each other by a system of pipes. The technical characteristics of the stations and the distances between them imply different costs of transporting of the water between the stations. The piping system and the costs for pumping one  $\text{m}^3$  on each section are shown in the Fig.40.

During a certain day, the stations  $P_1, P_3$  and  $P_6$  have  $a_1 = 30 \text{ m}^3$ ,  $a_3 = 30 \text{ m}^3$ ,  $a_6 = 40 \text{ m}^3$  of water, and the stations  $P_2, P_4$  and  $P_5$  require  $b_2 = 32 \text{ m}^3$ ,  $b_4 = 35 \text{ m}^3$ ,  $b_5 = 33 \text{ m}^3$  of water.

Determine a transport plan of the water between the six pumping stations optimal in terms of the required cost.

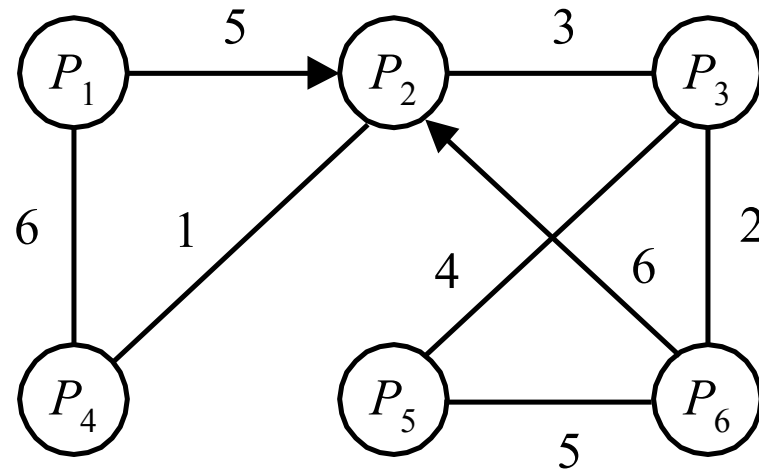



Fig.40. Pipeline network between pumping stations

 The statement problem is transfer type. It is considered that each pumping station can be both source and destination. Thus it extends the transfer problem to a minimizing transportation problem with 6 sources and 6 destinations.

The sum of the quantities available from the stations

$P_1$ ,  $P_3$  and  $P_6$  is equal to the sum of the quantities required at the stations  $P_2$ ,  $P_4$  and  $P_5$ , namely  $Q = 100 \text{ m}^3$ , so the problem is balanced.

We will enter the data of the transportation problem in a table (see Fig.41). The routes located on the diagonal of the table have zero unit costs; the blocked routes (between stations that are not connected by direct pipelines) have unit costs equal to  $M$  (where  $M \gg 0$ ). The available quantities are equal to  $a_i + Q$  ( $i = 1, \dots, 6$ ), and the required

From \ To	P 1	P 2	P 3	P 4	P 5	P 6	Supply
P 1	0	5	M	6	M	M	130
P 2	M	0	3	1	M	M	100
P 3	M	3	0	M	4	2	130
P 4	6	1	M	0	M	M	100
P 5	M	M	4	M	0	5	100
P 6	M	6	2	M	5	0	140
Demand	100	132	100	135	133	100	

Fig.41. The table of the transportation problem



quantities are equal to  $b_j + Q$  ( $j = 1, \dots, 6$ ).

Two optimal solutions are obtained (see Fig.42 and Fig.43).

The routes  $(i, i)$  are not considered.

The first optimal solution is:  $x_{14} = 30 \text{ m}^3$ ;  $x_{24} = 5 \text{ m}^3$ ;  
 $x_{32} = 37 \text{ m}^3$ ;  $x_{63} = 7 \text{ m}^3$ ;  $x_{65} = 33 \text{ m}^3$ .

-24-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	P 1	P 1	100	0	0	0
2	P 1	P 4	30	6	180	0
3	P 2	P 2	95	0	0	0
4	P 2	P 4	5	1	5	0
5	P 3	P 2	37	3	111	0
6	P 3	P 3	93	0	0	0
7	P 4	P 4	100	0	0	0
8	P 5	P 5	100	0	0	0
9	P 6	P 3	7	2	14	0
10	P 6	P 5	33	5	165	0
11	P 6	P 6	100	0	0	0
	Total	Objective	Function	Value =	475	

Fig.42. The first optimal solution of the transportation problem



2-24-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	P 1	P 1	100	0	0	0
2	P 1	P 2	30	5	150	0
3	P 2	P 2	65	0	0	0
4	P 2	P 4	35	1	35	0
5	P 3	P 2	37	3	111	0
6	P 3	P 3	93	0	0	0
7	P 4	P 4	100	0	0	0
8	P 5	P 5	100	0	0	0
9	P 6	P 3	7	2	14	0
10	P 6	P 5	33	5	165	0
11	P 6	P 6	100	0	0	0
	<b>Total</b>	<b>Objective</b>	<b>Function</b>	<b>Value =</b>	<b>475</b>	

Fig.43. The second optimal solution of the transportation problem

The second optimal solution is:  $x_{12} = 30 \text{ m}^3$ ;  $x_{24} = 35 \text{ m}^3$ ;  $x_{32} = 37 \text{ m}^3$ ;  $x_{63} = 7 \text{ m}^3$ ;  $x_{65} = 33 \text{ m}^3$ .

The value of the total transportation cost is equal to 475 m.u. 