**Partial Differential Equations**

**Part 1: Overview and Classification**

**Part 2: The Cauchy Problem**

**Part 3: Boundary Value Problems for PDE’s**

**Textbook:**

L. Debnath- Nonlinear Partial Differential Equations for Scientists and Engineers, Springer, 2012

**Classical Textbooks:**

1. Weinberger, H. F. – A First Course in Partial Differential Equations with Complex Variables and Transform Methods, Dover 1965.

2. John, F., - Partial Differential Equations, Springer 1971.

**Foreward**

* **The study of partial differential equations is an advanced topic that goes beyond an intermediate level course.**
* **Special methods are needed for each type of partial differential equation, even for each type of boundary conditions**
* **The notes below aim to give an idea about the problems involved and the methods that are applied.**
* **In Part1, we give the basic definition, discuss the notion of linearity, superposition principle and linear operators**
* **In Part 2, we define the Cauchy problem on the propogation oof initial datafor hyperbolic equations**
* **In Part 3, we overview basic results for boundary value problems of elliptic equations.**

**PART 3**

**Boundary Value Problems for PDE’s**

**3. Boundary- Value Problems**

Finding a function that solves a given partial differential equation and specific boundary conditions is known mathematically as solving a boundary-value problem. These problems are relevant for elliptic equations.

The second-order partial differential equation of the elliptic type with independent variables is of the form

where

Instead of dealing with general elliptic partial differential equations, we will start by presenting the simplest boundary problem value for the two-dimensional Laplace equation.

**Laplace’s equation:**

The solutions of the Laplace’s equation are closely related to complex analytic functions. Real and complex parts of complex analytic functions are solutions of the Laplace’s equation. Recall that the real and imaginaty parts of complex analytic functions satisfy the Cauchy-Riemann equations. By differentiating these equations and eliminating u or v one gets the Lapllace’s equation.

**Harmonic Function:**

A function is called a *harmonic* if it satisfies Laplace's equation in domain D and the first two derivatives are continuous in D.

Here, we can state that a linear combination of harmonic functions is harmonic, since Laplace's equation is linear and homogeneous.

**3.1 The First Boundary- Value Problem**

**The Dirichlet Problem:**

Find a harmonic function in satisfying the equation

on such that is a predetermined continuous function on the boundary of the field . is the interior of a simple closed segment smooth curve.

The solution to the Dirichlet problem can be interpreted as the steady-state temperature distribution in an object that does not physically contain source or any sinks of heat, with predicted temperature at all points on the boundary.

**3.2 The Second Boundary- Value Problem**

**The Neumann Problem:**

Find a harmonic function in satisfying the equation

on , and from this equation and Laplace equation,

on *The compatibility function*

The solution of the Neumann problem can be interpreted as the steady-state temperature distribution in an object that does not physically contain source or any sinks of heat, when the heat flux across the boundary is predicted.

Here the compatibility condition may be interpreted physically as the heat requirement that the net heat flux across the boundary be zero.

**3.3 The Maximum Principle**

Assume that is continuous in and harmonic in a bounded domain . The boundary of is where then reaches its maximum value.

This can be understood to mean that the electrostatic potential in a region without any free charge reaches its maximum (and minimum) values at the region's boundary and that the temperature of a body that was neither a source of heat nor a sink of heat acquires its largest maximum (and minimum values on the surface of the body.

We continue by listing a number of basic theorems:

**3.4 The Minimum Principle Theorem**

When is continuous in and harmonic in a bounded domain , it reaches its minimum on the border of .

**3.5 Uniqueness Theorem**

If there is a solution to the Dirichlet problem, it is unique.

**3.6 Continuity Theorem**

Dirichlet's solution the problem constantly depends on the boundary data.

**Overview of the notes:**

Part 1 of these lecture notes started by giving general information about Partial Differential Equations. Differential operators, the conditions of being a linear operator, The principle of linear superposition, and Hyperbolic, Parabolic, Elliptic Type Canonical Forms are given.In Part 2, Cauchy Problem definition and Cauchy-Kowalewskaya Theorem are detailed. In Part 3, Boundary-Value Problem, The First Boundary- Value Problems, The Second Boundary- Value Problems are given, and some important theorems are mentioned.

* Partial differential equations are those differential equations that involve more than one independent variable.

Differential equation:

Domain:

Initial Condition:

Boundary Conditions:

* A mathematical problem has to be “well posed”, this requires the **existence, uniqueness, continuity.**
* **Canonical Forms:**

Hyperbolic Type: ,

Parabolic Type:

Elliptic Type: