**Ordinary Differential Equations**

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**Textbook:**

1. Elementary Differential Equations and Boundary Value Problems.Ed8,BoyceR.C.DiPrima.JWS

**Foreward:**

* These lecture notes aim to review the material of the standard 2nd year differential equations course of engineering curricula and supplement it with an introduction to the study of partial differential equations.
* Ordinary differential equations are used in many engineering applications.
  + **First order ODE’s** describe many growth and decay phenomena.
  + **Second order ODE’s** describe osciallatory phenomena.
  + **Series solutions** are needed for the study of for example Bessel funcions that describe propagation of waves.
  + The method of **Laplace transform** is used in taking into account initial conditions.
  + **Systems of ODE’s** are used as models for all linear systems and find important applications in control theory. re are many fine textbooks part on ordinary differential equations is based
* Partial differential equations solvable by the method of separation of variables are also studied in a first course in differential equations. These topics are presented under the PDE headings.

**The notes on ODE’S consist of the following parts**

* Part 1: First order ordinary differential equations
* Part 2: Higher order ordinary differential equations
* Part 3: Series solutions
* Part 4: Laplace transform
* Part 5: Systems of ODE’s

**Part 2**

**Higher Order Ordinary Differential Equations**

**Second order homogeneous equations with constant coefficients.**

These equations are of the form

We put where r is constant .

We compute,

We substitute in the differential equation to get

We divide by to get the characteristic eqn.

whose roots are

If we get two distinct roots

In this case, there one 2 **linearly independent** solutions

and the **general solution** is given by

where and are constants.

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| **Ex:**  You should solve almost all of the ‘’easy’’ problems of section 3.1. |

**Fundamental solution of the linear homogeneous equation:**

We consider second order homogeneous equations whose coefficients are not necessarily constant.

The initial conditions are specified at ,

This problem has a unique solution on any interval on which are continuous

If , ie

Then, any linear combination of the solution is again a solution

* Two solutions, are linearly independent if
* If , , then

is called a fundamental set of solutions.

**Linear independence and Wronskian:**

Two function and are called **linearly independent** if the equation

**(1)**

implies .

If this equation is zero ,then its derivative is also zero:

. **(2)**

**Ex:**

We can express (1) and (2) as a matrix equation.

Let W(t)= be the determinant of the coefficient matrix . If even at a single point , then .

If f and g are solutions of a second order equation

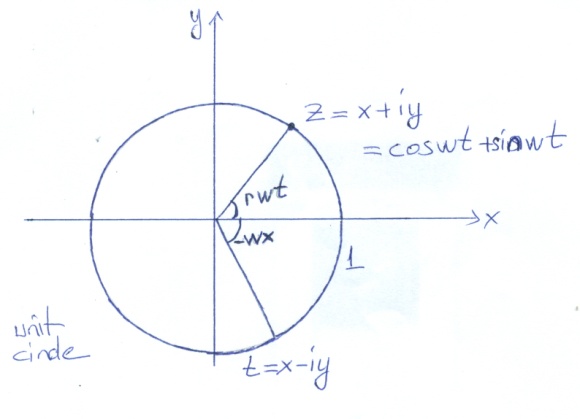
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then is either identically zero or it is never zero.

**Complex roots of the characteristic equation**

If is negative then the roots of the characteristic equation are complex:

We denote , and we use



If the roots are complex, we write them as

And the general solutions

If you find and ,you can write the solution directly as

, where

**Ex.:**

**Ex:** ,

With these methods, one can solve almost all of the ‘’easy’’ problems.

**Repeated roots , reduction of order**

If the roots of the characteristic equation are repeated roots ,i.e., then ,one solution is

And the other solution is

We can check this in an example.

**Ex:**

We can prove that as follows, using **the method of reduction of order**

Let where is a new function.

hence

Since , we can take hence

**Nonhomogeneous equations: The method of undetermined coefficients**

We consider equations of the form

**(1)**

If is a linear combination of

**(2)**

then we can use **the method of undetermined coefficients**

A particular solution of (1) is found by this method as follows;

Let and be the roots of the characteristic equation.

* If and

then we substitute in (1) and find A.

* If g(t) is a sum of such terms ,i.e.,

then

* If then provided that are not among the solution of the homogeneous equation.
* We will give a general method after we learn about higher order equations.
* If is not of the form (2) ,we should use the method of variation of parameters.

**General theary of higher order equations**

The solution of the homogeneaus equation with constant cofficients is by starting with

differentiating and substituting in the differential equation, to get the characteristic equation.

* The n’th order equation admits n initial conditions
* The general solution is a linear conbination of n linearly independent solutions.

**Homogeneous equations with constant coefficients**

**Problem 1:** Given the differential equation ,find the roots of the characteristic equations

**Ex**

**(1)**

**Remark** The rational roots divide 6. Try

is a solution, r=-1 is also a solution should divide (1)

Solve

**Problem 2** We may give you directly the characteristic equation and ask you to find the solution

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| **Ex:** | |
| **Ex:** | |

* If there in a repeated root of order n, we add

|  |
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| **Ex:** |

**The method of operators** for solving nonhomogeneous equations. This method is valid only for equations with **constant coefficients:**

Let

Consider for example,

.

In practice, we just replace r by D in the charactristic eqn and apply it to y:

In order to solve the inhomogeneous equation, we note that we apply to both side

The undetermined coefficient A has to be found from the equation

This method is particularly useful if has tems that are also solutions of the homogeneous equation:

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| **Ex:** |

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| **Ex:** |

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| **Ex:** |

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| **Ex:** |

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| **Ex:** |

**END OF PART 2**