ThinkBS 309 Markov Chains and Its Applications (Advanced)

Offered in Debreceni Egyetem

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Course Objectives:

To obtain an advanced understanding on linear algebraic and numerical methods in the theory of Markov chains, with a wide variety of applications in Mathematics, Physics, Engineering, Economics and Computer Science.

Course Contents:

Advancved methods in linear algebra, graph theory, Markov chains. Ergodic and absorbing Markov chains, stationary distribution, recurrence, absorption, reversibility, mixing time.

Learning Outcomes of the Course Unit:

1. To model and solve problems in engineering using linear algebraic techniques.
2. To utilize numerical methods and other algorithms.
3. To study real-life problems in an abstract framework, in a systematic way.

Bibliography:

WEEKLY SUBJECTS AND RELATED PREPARATION STUDIES:

Week Subjects

1 Linear algebraic background. Eigenvalues, eigenvectors. Jordan- and Frobenius normal form. Matrix multiplication, exponentiation.

2 Effective deterministic and non-deterministic numerical methods (significantly faster than Gaussian elimination) to matrix product, exponentiation, rank, inverse, normal forms, and to the solution of systems of linear equations.

3 Graphs, adjacency matrices, spectra. Cauchy interlacing theorem. Graph properties determined by the spectrum.

4 Perron-Frobenius theorem, Gershgorin circles, numerical methods.

5 Finite Markov chains as digraphs, transition matrix. Basic notions: irreducibility, regularity, absorption, ergodicity, recurrence. Some applications.

6 Irreducible and regular Markov chains. Stationary distribution and its computation, examples.

7 Frequency of recurrence, law of large numbers and central limit theorem in irreducible Markov chains.

8 Absorbing Markov chains. Basic formulas: probability of absorption in each state, expected value and higher moments of the time to absorption.

9 Reversible chains. Kolmogorov criterion. Related algorithms and numerical approximations.

10 Mixing time. Random walks on simple, undirected graphs. Expanders. Pseudo-randomness. Applications, examples.

11 Nondeterministic decision-making and other evolutionary processes. Applications in computer science.

12 Countably infinite Markov chains.