

### Markov Chains and Their Applications, Problem sheet 8

- (1) By using the Jordan normal form, prove that if  $A^k$  converges for a real matrix  $A$ , then the convergence must have exponential speed.
- (2) Conclude from Problem 1 that  $P^n$  converges to  $W$  with exponential speed if  $P$  is a regular Markov chain.
- (3) Show that  $PW = WP = W$ , and  $(P - W)^n = P^n - W$ .
- (4) Prove that  $Z\underline{1} = \underline{1}$ ,  $\underline{w}^*Z = \underline{w}^*$ , and  $Z(I - P) = I - W$ .
- (5) Compute the fundamental matrix for the regular chains seen before during the course.
- (6) Compute the mean recurrence times, mean passage times and the  $\sigma_i$  as a continuation of Problem 5.