Markov Chains and Their Applications, Problem sheet 6

(1) Prove that given an irreducible matrix A and index i, the numbers k such that $(A^k)[i,i] > 0$ form a sequence that is eventually arithmetic: that is, there is a large enough K and number p, r such that for $m > K$, an exponent is good iff it is of the form $p\ell + r$.

(2) By using a density argument, prove that p is the same for all indices i. Conclude that every vertex has the same period.

(3) Find an irreducible, aperiodic chain such that the (square) zero-matrices along the diagonal in the "canonical" form PAP^{-1} are of different sizes, the non-zero blocks A_j are square, and the period p does not divide n.