Markov Chains and Their Applications, Problem sheet 5

- (1) Show that if P is the transition matrix of a Markov chain, then all Gershgorin discs of P have a common point.
- (2) Prove the proposition about $P^m[i, j]$.
- (3) Show that a Markov chain is absorbing iff all sinks of the induced digraph on its strong components are singletons.
- (4) Show that a matrix P is a transition matrix (of some Markov chain) iff P is non-negative and $P\underline{1} = \underline{1}$.
- (5) What is the difference between acyclic graphs and partial orders?
- (6) Prove that if $P\hat{A}$ is absorbing, then so is P^m for any m. What does an absorbing, primitive chain look like?