Markov Chains and Their Applications, Problem sheet 4

(1) Show that the inverse of a stochastic matrix A is non-negative iff it is stochastic. Prove that if this is the case, then A is a permutation matrix.

(2) Generalize the previous exercise to non-negative (invertible) matrices.

(3) By using the Jordan normal form of the matrix $P = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$, find the formula for the elements of P^n.

(4) Observe that the series of vectors $\underline{u}_n = (F_n, F_{n+1})^*$, where F_n is the n-th Fibonacci number, satisfies the relations $\underline{u}_0 = (0,1)^*$ and $\underline{u}_n = P\underline{u}_{n-1}$, with P as in the previous exercise. Deduce that $\underline{u}_n = P^n\underline{u}_0$. By the previous exercise, find the exact formula for F_n.

(5) Show that the union of the Gershgorin discs coincide with the spectrum iff the matrix is diagonal.