

Markov Chains and Their Applications, Problem sheet 2

- (1) Multiply the matrices $A = \begin{pmatrix} 2 & -3 \\ 1 & 7 \end{pmatrix}$ and $B = \begin{pmatrix} -5 & 6 \\ 9 & 8 \end{pmatrix}$ via the Strassen algorithm. Observe that it only requires 7 multiplications.
- (2) Show that the companion matrix A of a polynomial $f(x)$ has minimal and characteristic polynomial $m_A(x) = \chi_A(x) = f(x)$.
- (3) Using the Frobenius normal form and companion matrices, construct a matrix A with minimal polynomial $m_A(x) = x^2(x^2 + 1)(x - 2)$ and characteristic polynomial $\chi_A(x) = x^3(x^2 + 1)^2(x - 2)$. In particular, observe that the minimal polynomial of a matrix is not necessarily irreducible.
- (4) Compute the inverse of the matrix constructed in problem 3. Show that the computation runs in linear time for a Frobenius normal form.
- (5) Compute the cube of the matrix constructed in problem 3.
- (6) What is $\dim \ker(A)$ for the matrix A constructed in problem 3?