Markov Chains and Their Applications, Problem sheet 12

(1) Compute the fundamental matrix of the drunkard walk (fair gambler's ruin).

(2) Compute the first four moments of the gambler's ruin.

(3) Find and verify the formulas for the probabilities of absorption and the expected runtime in the unfair gambler's ruin problem, when the probability to move to the left in each transient state is a fixed $p > 1/2$. (And understand why it is important to shuffle the deck properly before a new game of blackjack or poker.)

(4) Show that the fair gambler's ruin is indeed a fair game, i.e., a martingale, whereas the unfair gambler's ruin in the previous problem is a supermartingale.