

## Markov Chains and Their Applications, Problem sheet 11

- (1) Show that the total variation distance from any distribution  $\underline{w}^*$  attains its maximum on a Dirac measure.
- (2) Prove that the number of  $(p, q)$  riffle shuffles (when the deck of  $N$  cards is cut into piles of size  $p$  and  $q$ ) is  $\binom{p+q}{q}$ .
- (3) Using the result of Problem 2, show that the total number of riffle shuffles is  $2^N - N$ . (Watch out for the identity permutation!)
- (4) Prove that the transition matrix of a random walk on a simple, connected undirected graph is symmetrical iff the graph is regular.
- (5) Prove that the stationary distribution of the transition matrix of a random walk on a simple, connected undirected graph is  $\underline{1}^*$  iff the graph is regular.