

Markov Chains and Their Applications, Problem sheet 10

- (1) Prove that if P is irreducible, then so is \widehat{P} .
- (2) Show that $\widehat{\widehat{P}} = P$.
- (3) Prove that if $P = P^*$ then P is reversible. (What is \underline{w} ?)
- (4) Construct a reversible P that is not symmetric with the smallest n possible.
- (5) Let $V = \{x_1, \dots, x_n, y_1, \dots, y_n\}$ be the vertices of a graph with edges $x_i x_{i+1}, x_i y_{i+1}, y_i x_{i+1}, y_i y_{i+1}$ (put $x_{n+1} = x_1$ and $y_{n+1} = y_1$). Show that there are at least 1.414^n minimal cycles in this graph.
- (6) Construct a graph and improve the lower bound to 1.442^n .