Markov Chains and Their Applications, Problem sheet 1

(1) Are the given vectors independent? In case they are, then express one of the vectors as a linear combination of the others.
  (a) $(1, 2, -7)^T, (-3, 4, -3)^T, (-1, 8, -17)^T$
  (b) $(-2, 2, -3)^T, (6, 7, 9)^T, (6, -1, 11)^T$
  (c) $(5, 3, -10)^T, (-2, 3, -9)^T, (-3, 4, -5)^T$
  (d) $(1, 1, 1)^T, (1, 2, 2)^T, (3, 4, 4)^T$

(2) Consider the four subspaces generated by the system of vectors in each subproblem of the previous exercise. Which of them contain the vector $\underline{b} = (0, 10, -24)^T$?

(3) Find the change of basis matrix for converting from the standard basis to the following bases.
  (a) $\underline{f_1} = (2, -1, 0), \underline{f_2} = (1, 0, 1), \underline{f_3} = (0, 0, -2)$
  (b) $\underline{h_1} = (1, 1, 1), \underline{h_2} = (1, 1, 0), \underline{h_3} = (2, 1, 1)$
  What is the change of basis matrix for converting from (f) to (g)? And that from (g) to (f)?

(4) Solve the system of linear equations.

(a)
$$-3x_1 + 4x_2 - x_3 + 2x_4 = 0$$
$$x_1 + 2x_2 - 4x_3 + 5x_4 = -11$$
$$5x_1 - 5x_2 \phantom{{}+{}} + \phantom{{}} 2x_4 = -1$$
$$-4x_1 - 3x_2 + 10x_3 - 9x_4 = -3$$

(b)
$$x_1 + 2x_2 - 3x_3 = 8$$
$$-2x_1 - 5x_2 - 4x_3 = -8$$
$$4x_1 + 7x_2 + x_3 = 17$$
$$2x_1 + 9x_2 + 21x_3 = -1$$

(c)
$$-x_1 + 4x_2 - x_3 + 2x_4 = 1$$
$$x_1 + 2x_2 - 5x_3 + 4x_4 = -13$$
$$7x_1 - 5x_2 + 6x_3 + 2x_4 = -11$$
$$4x_1 + x_2 + 9x_3 + 2x_4 = 4$$

(d)
$$5x_1 + 4x_2 - 6x_3 + 3x_4 = -3$$
$$3x_1 + 2x_2 - 5x_3 + 6x_4 = -19$$
$$2x_1 + 2x_2 - x_3 - 3x_4 = 11$$

(5) Consider the systems of linear equations in the previous problem that had at least one solution. Where there is exactly one solution and the matrix of coefficients on the left-hand side is square, compute the determinant and inverse of the square matrix. When you obtain infinitely many solutions, compute the rank of the matrix and compare it to the dimension of the affine subspace of all solutions (i.e., the number of free variables).