Artificial Intelligence
Chapter 7

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reorganized by L. Aszalós

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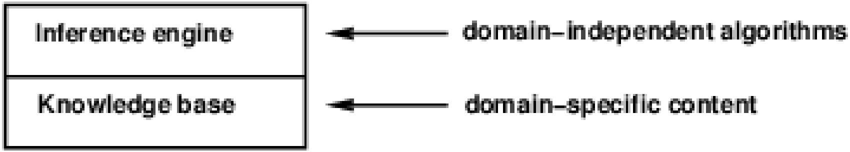
Outline

- Knowledge-based agents
- Wumpus world
- Logic in general—models and entailment
- Propositional (Boolean) logic
- Equivalence, validity, satisfiability
- Inference rules and theorem proving
  - forward chaining
  - backward chaining
  - resolution

Knowledge bases



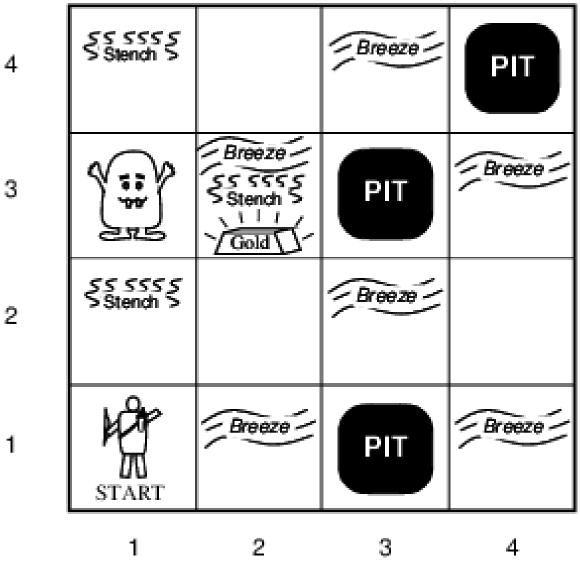
- **Knowledge base** = set of **sentences** in a *formal* language
- **Declarative** approach to building an agent (or other system):
  - `Tell` it what it needs to know
- Then it can `Ask` itself what to do—answers should follow from the KB
- Agents can be viewed at the **knowledge level**
  - i.e., *what they know*, regardless of how implemented
- Or at the **implementation level**
  - i.e., data structures in KB and algorithms that manipulate them

A simple knowledge-based agent

```
function KB-Agent(percept): an action
    static: KB, a knowledge base
            t, a counter, initially 0, indicating time
    Tell(KB, Make-Percept-Sentence(percept))
    action := Ask(KB, Make-Action-Query(t))
    Tell(KB, Make-Action-Sentence(action, t))
    t := t + 1
    return action
```

- The agent must be able to:
  - Represent states, actions, etc.
  - Incorporate new percepts
  - Update internal representations of the world
  - Deduce hidden properties of the world
  - Deduce appropriate actions

Wumpus world



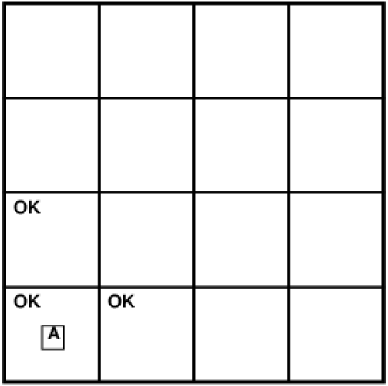
Wumpus World PEAS description

- Performance measure
  - gold +1000, death -1000, -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square
- Actuators
  - Left turn, Right turn, Forward, Grab, Release, Shoot
- Sensors
  - Breeze, Glitter, Smell

Wumpus world characterization

- Observable
  - No—only *local* perception
- Deterministic
  - Yes—outcomes exactly specified
- Episodic
  - No—sequential at the level of actions
- Static
  - Yes—Wumpus and Pits do not move
- Discrete
  - Yes
- Single-agent
  - Yes—Wumpus is essentially a natural feature

Exploring a wumpus world



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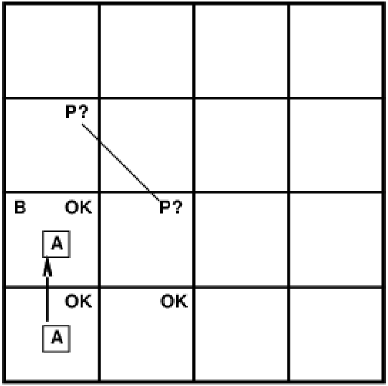

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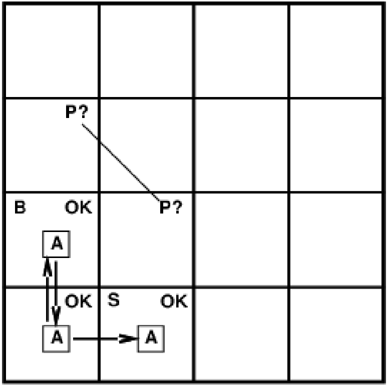

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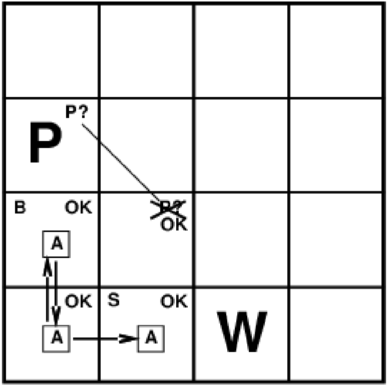

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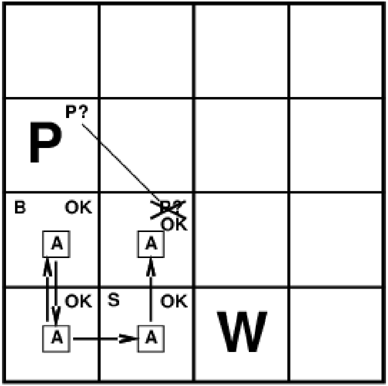

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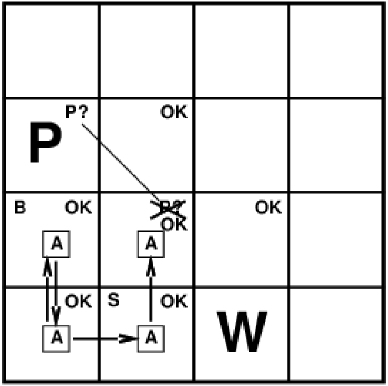

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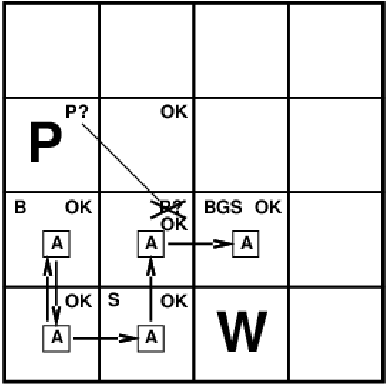
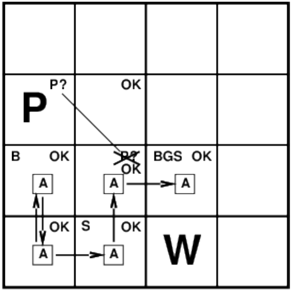


Other tight spots

- Breeze in (1,2) and (2,1) ⇒ no safe actions
- Assuming pits uniformly distributed, (2,2) has pit w/ prob 0.86, vs. 0.31



- Smell in (1,1) ⇒ cannot move
- Can use a strategy of **coercion**
  - shoot straight ahead
    - wumpus was there ⇒ dead ⇒ safe
    - wumpus wasn't there ⇒ safe

Logic in general

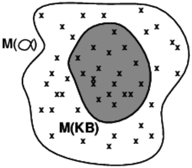
- **Logics** are formal languages for representing information such that conclusions can be drawn
- **Syntax** defines the sentences in the language
- **Semantics** define the "meaning" of sentences;
  - i.e., define **truth** of a sentence in a world
- E.g., the language of arithmetic
  - $x + 2 \geq y$ is a sentence; $x2 + y >$ is not a sentence
  - $x + 2 \geq y$ is true iff the number $x + 2$ is no less than the number y
  - $x + 2 \geq y$ is true in a world where $x = 7,\ y = 1$
  - $x + 2 \geq y$ is false in a world where $x = 0,\ y = 6$

Entailment

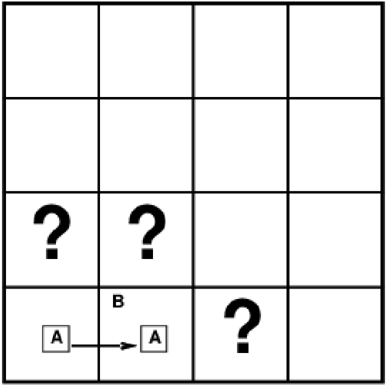
- **Entailment** means that one thing *follows from* another: $KB \models \alpha$
- Knowledge base KB entails sentence α if and only if
  - α is true in all worlds where KB is true
- E.g., the KB containing *the Giants won* and *the Reds won* entails *Either the Giants won or the Reds won*
- E.g., $x + y = 4$ entails $4 = x + y$
- Entailment is a relationship between sentences (i.e., *syntax*) that is based on *semantics*
- Note: brains process *syntax* (of some sort)

Models

- Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated
- We say *m is a model of* a sentence α if α is true in m
- $M(\alpha)$ is the set of all models of α
- Then $KB \models \alpha$ if and only if $M(KB) \subseteq M(\alpha)$
- $KB =$ "Giants won and Reds won", $\alpha =$ "Giants won"

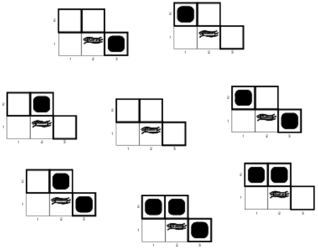


Entailment in the wumpus world

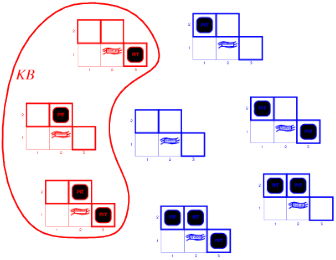
- Situation after detecting nothing in [1,1],
- moving right, breeze in [2,1]
- Consider possible models for ?s assuming only pits
- 

- 3 Boolean choices ⇒ 8 possible models

Wumpus models

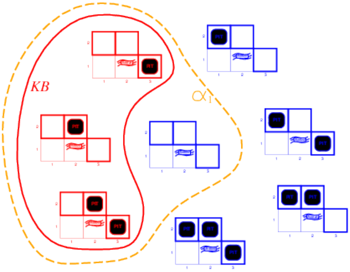


Wumpus models



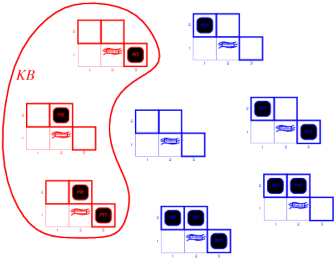
- KB = wumpus-world rules + observations

Wumpus models



- KB = wumpus-world rules + observations
- α_1 = "[1,2] is safe", $KB \models \alpha_1$, proved by **model checking**

Wumpus models



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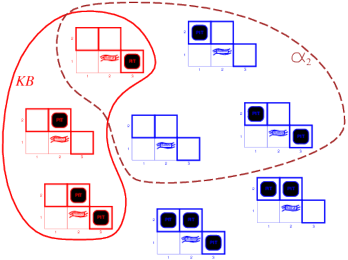
Wumpus models



- KB = wumpus-world rules + observations
- α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference

- $KB \vdash_i \alpha$ = sentence α can be derived from KB by procedure i
- Consequences of KB are a haystack; α is a needle.
  - Entailment = needle in haystack; inference = finding it
- **Soundness**: i is sound if
  - whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$
- **Completeness**: i is complete if
  - whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$
- Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.
- That is, the procedure will answer any question whose answer follows from what is known by the KB.

Propositional logic: Syntax

- Propositional logic is the simplest logic—illustrates basic ideas
- The proposition symbols P_1, P_2 etc are sentences
  - if S is a sentence, $\neg S$ is a sentence (**negation**)
  - if S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (**conjunction**)
  - if S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (**disjunction**)
  - if S_1 and S_2 are sentences, $S_1 \supset S_2$ is a sentence (**implication**)
  - if S_1 and S_2 are sentences, $S_1 \equiv S_2$ is a sentence (**biconditional**)

Propositional logic: Semantics

- Each model specifies true/false for each proposition symbol
- E.g. $P_{1,2}$=true, $P_{2,2}$=true, $P_{3,1}$=false
  - With these symbols, 8 possible models, can be enumerated automatically.
- Rules for evaluating truth with respect to a model m:
-
$\neg S$	is true iff	S	is false		
$S_1 \wedge S_2$	is true iff	S_1	is true *and*	S_2	is true
$S_1 \vee S_2$	is true iff	S_1	is true *or*	S_2	is true
$S_1 \supset S_2$	is true iff	S_1	is false *or*	S_2	is true
i.e.,	is false iff	S_1	is true *and*	S_2	is false
$S_1 \equiv S_2$	is true iff	$S_1 \supset S_2$	is true *and*	$S_2 \supset S_1$	is true

- Simple recursive process evaluates an arbitrary sentence, e.g.,
  - $\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{false} \vee \mathit{true}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$

Truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \supset Q$	$P \equiv Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
true	false	false	false	true	false	false
true	true	false	true	true	true	true

Wumpus world sentences

- Let $P_{i,j}$ be true if there is a pit in $[i,j]$.
- Let $B_{i,j}$ be true if there is a breeze in $[i,j]$.
  - $\neg P_{1,1}$
  - $\neg B_{1,1}$
  - $B_{2,1}$
- "Pits cause breezes in adjacent squares"
  - $B_{1,1} \equiv (P_{1,2} \vee P_{2,1})$
  - $B_{2,1} \equiv (P_{1,1} \vee P_{2,2} \vee P_{3,1})$
- "A square is breezy *if and only if* there is an adjacent pit"

Truth tables for inference

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	R_1	R_2	R_3	R_4	R_5	KB
false	false	false	false	false	false	false	true	true	true	true	false	false
false	false	false	false	false	false	true	true	true	false	true	false	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
false	true	false	false	false	false	false	true	true	false	true	true	false
false	true	false	false	false	false	true	true	true	true	true	true	*true*
false	true	false	false	false	true	false	true	true	true	true	true	*true*
false	true	false	false	false	true	true	true	true	true	true	true	*true*
false	true	false	false	true	false	false	true	false	false	true	true	false
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
true	true	true	true	true	true	true	false	true	true	false	true	false

- Enumerate rows (different assignments to symbols),
  - if *KB* is true in row, check that α is too

Inference by enumeration

- Depth-first enumeration of all models is sound and complete

```
function TT-Entails?(KB,alpha): true or false
    inputs: KB, the knowledge base, a sentence in propositional logic
            alpha, the query, a sentence in propositional logic

    symbols := a list of the proposition symbols in KB and alpha
    return TT-Check-All(KB, alpha, symbols, [])

function TT-Check-All(KB, alpha, symbols, model): true or false
    if Empty?(symbols) then
          if PL-True?(KB, model) then return PL-True?(alpha, model)
          else return true
    else do
          P := First(symbols);
          rest := Rest(symbols)
          return TT-Check-All(KB, alpha, rest, Extend(P, true, model)) and
                 TT-Check-All(KB, alpha, rest, Extend(P, false, model}))
```

- $O(2^n)$ for n symbols; problem is *co-NP-complete*

Logical equivalence

Two sentences are **logically equivalent** iff true in same models: $\alpha \Leftrightarrow \beta$ if and only if $\alpha \models \beta$ and $\beta \models \alpha$

$$
\begin{array}{rcll}
(\alpha \wedge \beta) & \Leftrightarrow & (\beta \wedge \alpha) & \text{commutativity of } \wedge \\
(\alpha \vee \beta) & \Leftrightarrow & (\beta \vee \alpha) & \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) & \Leftrightarrow & (\alpha \wedge (\beta \wedge \gamma)) & \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) & \Leftrightarrow & (\alpha \vee (\beta \vee \gamma)) & \text{associativity of } \vee \\
\neg(\neg\alpha) & \Leftrightarrow & \alpha & \text{double-negation elimination} \\
(\alpha \supset \beta) & \Leftrightarrow & (\neg\beta \supset \neg\alpha) & \text{contraposition} \\
(\alpha \supset \beta) & \Leftrightarrow & (\neg\alpha \vee \beta) & \text{implication elimination} \\
(\alpha \equiv \beta) & \Leftrightarrow & ((\alpha \supset \beta) \wedge (\beta \supset \alpha)) & \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) & \Leftrightarrow & (\neg\alpha \vee \neg\beta) & \text{De Morgan} \\
\neg(\alpha \vee \beta) & \Leftrightarrow & (\neg\alpha \wedge \neg\beta) & \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) & \Leftrightarrow & ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) & \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) & \Leftrightarrow & ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) &
\end{array}
$$

Validity and satisfiability

- A sentence is **valid** if it is true in *all* models,
  - e.g., *True*, $A \vee \neg A$, $A \supset A$, $(A \wedge (A \supset B)) \supset B$
- Validity is connected to inference via the **Deduction Theorem**:
  - $KB \models \alpha$ if and only if $(KB \supset \alpha)$ is valid
- A sentence is **satisfiable** if it is true in *some* model
  - e.g., $A \vee B$, C
- A sentence is **unsatisfiable** if it is true in *no* models
  - e.g., $A \wedge \neg A$
- Satisfiability is connected to inference via the following:
  - $KB \models \alpha$ if and only if $(KB \wedge \neg\alpha)$ is unsatisfiable
  - i.e., prove α by *reductio ad absurdum*

Proof methods

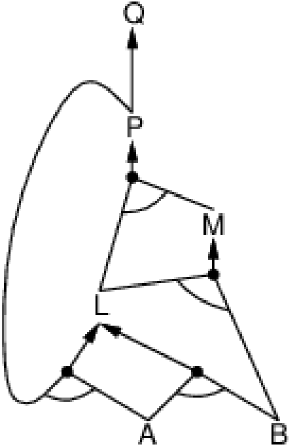
- Proof methods divide into (roughly) two kinds:
- **Application of inference rules**
  - Legitimate (sound) generation of new sentences from old
  - **Proof** = a sequence of inference rule applications
  - Can use inference rules as operators in a standard search alg.
  - Typically require translation of sentences into a **normal form**
- **Model checking**
  - truth table enumeration (always exponential in n)
  - improved backtracking, e.g., Davis–Putnam–Logemann–Loveland
  - heuristic search in model space (sound but incomplete)
  - e.g., min-conflicts-like hill-climbing algorithms

Forward and backward chaining

- **Horn Form** (restricted)
  - KB = *conjunction* of *Horn clauses*
  - Horn clause =
    - ★ proposition symbol; or
    - ★ (conjunction of symbols) $\supset$ symbol
  - E.g., $C \wedge (B \supset A) \wedge (C \wedge D \supset B)$
- **Modus Ponens** (for Horn Form): complete for Horn KBs
- $$\frac{\alpha_1, \ldots, \alpha_n, \qquad \alpha_1 \wedge \cdots \wedge \alpha_n \supset \beta}{\beta}$$
- Can be used with **forward chaining** or **backward chaining**.
  - These algorithms are very natural and run in *linear* time

Forward chaining

- Idea: fire any rule whose premises are satisfied in the KB, add its conclusion to the KB, until query is found



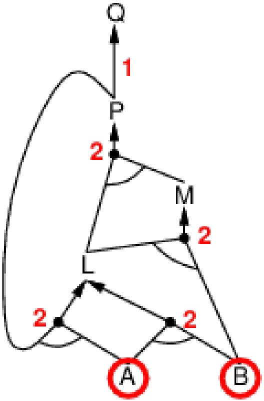
-
- $P \supset Q$, $L \wedge M \supset P$, $B \wedge L \supset M$, $A \wedge P \supset L$, $A \wedge B \supset L$, A, B

Forward chaining algorithm

```
function PL-FC-Entails?(KB,q): true or false
    inputs: KB, the knowledge base, a set of propositional Horn clauses
            q, the query, a proposition symbol
    local variables: count, a table, indexed by clause, initially the number of prem
                     inferred, a table, indexed by symbol, each entry initially fal
                     agenda, a list of symbols, initially the symbols known in KB

    while agenda is not empty do
        p := Pop(agenda)
        unless inferred[p] do
            inferred[p] := true
            for each Horn clause c in whose premise p appears do
                      decrement count[c]
                      if count[c] = 0 then do
                            if Head[c] = q then return true
                            Push(Head[c], agenda)
    return false
```

Forward chaining example



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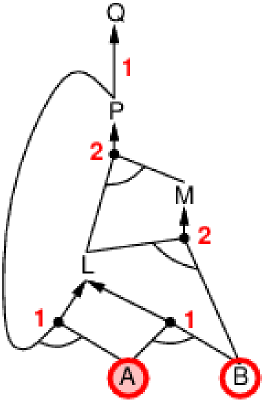

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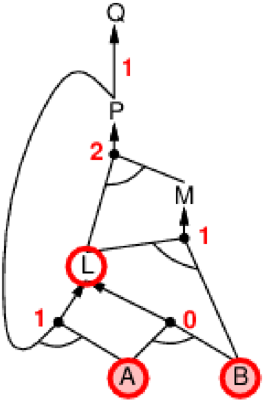

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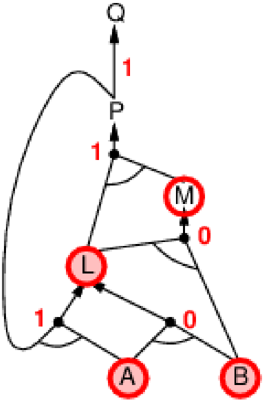

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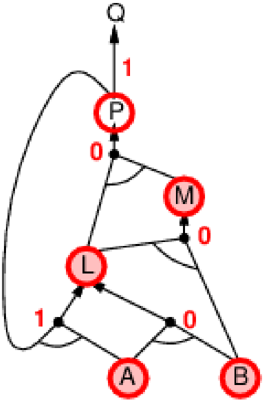

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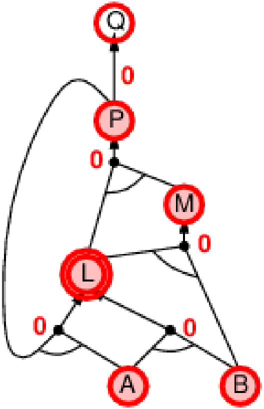

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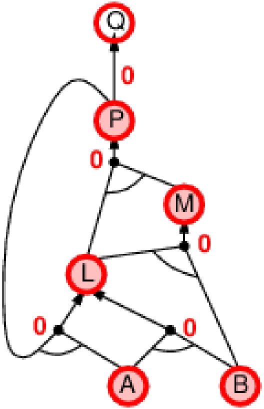

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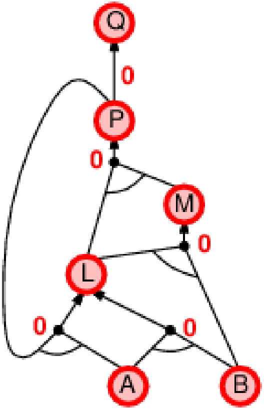


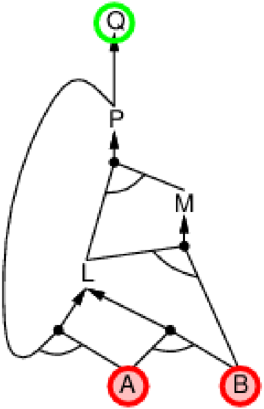
Proof of completeness

- FC derives every atomic sentence that is entailed by KB
  - FC reaches a **fixed point** where no new atomic sentences are derived
  - Consider the final state as a model m, assigning true/false to symbols
  - Every clause in the original KB is true in m
    - *Proof*: Suppose a clause $a_1 \wedge \ldots \wedge a_k \supset b$ is false in m
    - Then $a_1 \wedge \ldots \wedge a_k$ is true in m and b is false in m
    - Therefore the algorithm has not reached a fixed point!
  - Hence m is a model of KB
  - If $KB \models q$, q is true in *every* model of KB, including m
- *General idea*: construct any model of KB by sound inference, check α

Backward chaining

- Idea: work backwards from the query q:
  - to prove q by BC,
    - check if q is known already, or
    - prove by BC all premises of some rule concluding q
- Avoid loops: check if new subgoal is already on the goal stack
- Avoid repeated work: check if new subgoal
  - has already been proved true, or
  - has already failed

Backward chaining example



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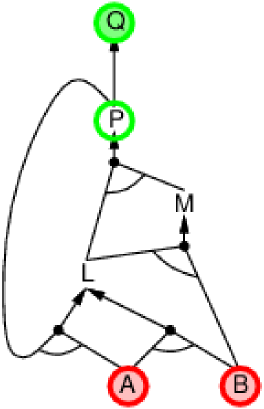

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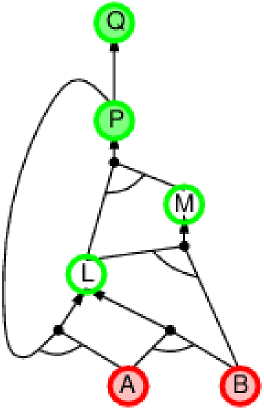
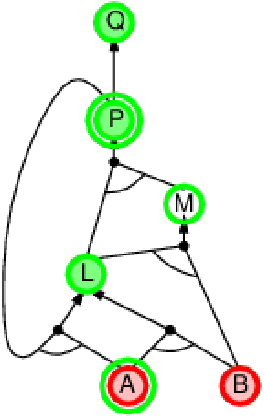
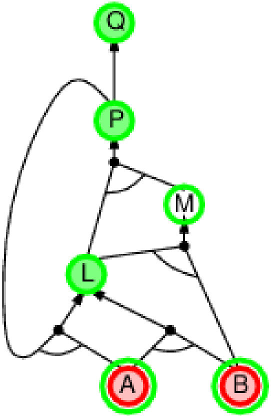


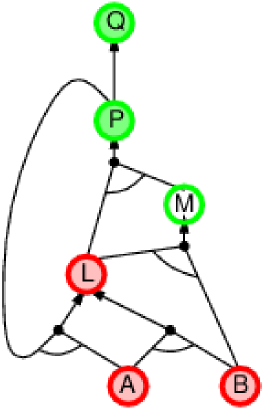
Backward chaining example



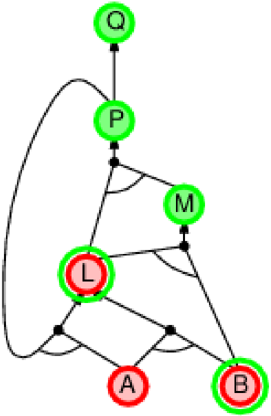
Backward chaining example



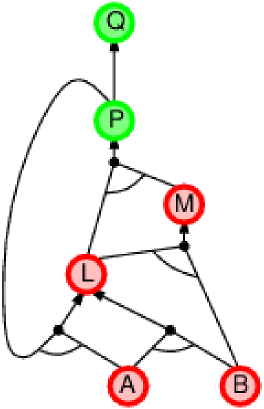
Backward chaining example



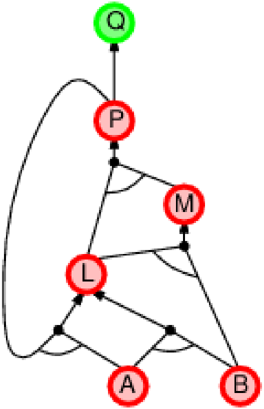
Backward chaining example



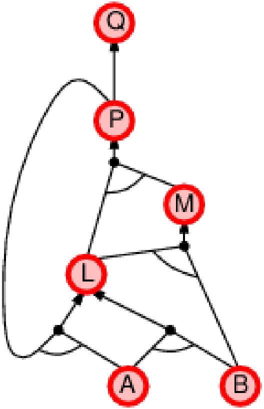
Backward chaining example



Backward chaining example



Backward chaining example



Forward vs.~backward chaining

- FC is **data-driven**, cf. automatic, unconscious processing,
  - e.g., object recognition, routine decisions
  - May do lots of work that is irrelevant to the goal
- BC is **goal-driven**, appropriate for problem-solving,
  - e.g., Where are my keys? How do I get into a PhD program?
  - Complexity of BC can be *much less* than linear in size of KB

Resolution

- **Conjunctive Normal Form** (CNF—universal)
  - *conjunction* of *disjunctions* of *literals*
  - clause = disjunction of literals
  - E.g., $(A \vee \neg B) \wedge (B \vee \neg C \vee \neg D)$
- **Resolution** inference rule (for CNF): complete for propositional logic
- $$\frac{\ell_1 \vee \cdots \vee \ell_k, \qquad m_1 \vee \cdots \vee m_n}{\ell_1 \vee \cdots \vee \ell_{i-1} \vee \ell_{i+1} \vee \cdots \vee \ell_k \vee m_1 \vee \cdots \vee m_{j-1} \vee m_{j+1} \vee \cdots \vee m_n}$$
- where ℓ_i and m_j are complementary literals. E.g.,
  - $$\frac{P_{1,3} \vee P_{2,2}, \qquad \neg P_{2,2}}{P_{1,3}}$$

*Resolution is sound and complete for propositional logic

Conversion to CNF

- $B_{1,1} \equiv (P_{1,2} \vee P_{2,1})$
  - Eliminate $\equiv$, replacing $\alpha \equiv \beta$ with $(\alpha \supset \beta) \wedge (\beta \supset \alpha)$.
  - $$(B_{1,1} \supset (P_{1,2} \vee P_{2,1})) \wedge ((P_{1,2} \vee P_{2,1}) \supset B_{1,1})$$
  - Eliminate $\supset$, replacing $\alpha \supset \beta$ with $\neg\alpha \vee \beta$.
  - $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg(P_{1,2} \vee P_{2,1}) \vee B_{1,1})$$
  - Move $\neg$ inwards using de Morgan's rules and double-negation:
  - $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge ((\neg P_{1,2} \wedge \neg P_{2,1}) \vee B_{1,1})$$
  - Apply distributivity law ($\vee$ over $\wedge$) and flatten:
  - $$(\neg B_{1,1} \vee P_{1,2} \vee P_{2,1}) \wedge (\neg P_{1,2} \vee B_{1,1}) \wedge (\neg P_{2,1} \vee B_{1,1})$$

Resolution algorithm

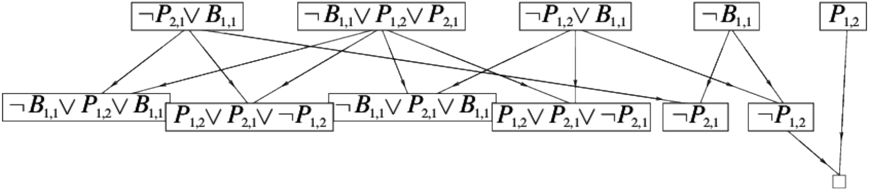
Proof by contradiction, i.e., show $KB \wedge \neg\alpha$ unsatisfiable

```
function PL-Resolution(KB, alpha): true or false
    input: KB, the knowledge base, a sentence in propositional logic
           alpha, the query, a sentence in propositional logic

    clauses := the set of clauses in the CNF representation of (KB and not alpha)
    new := {}
    loop do
        for each C_i, C_j in clauses do
            resolvents := PL-Resolve(C_i, C_j)
            if resolvents contains the empty clause then return true
            new := new union resolvents
        if new subset of clauses then return false
        clauses := clauses union new
```

Resolution example

$KB = (B_{1,1} \equiv (P_{1,2} \vee P_{2,1})) \wedge \neg B_{1,1}$, $\alpha = \neg P_{1,2}$



Summary

- Logical agents apply **inference** to a **knowledge base**
  - to derive new information and make decisions
- Basic concepts of logic:
  - **syntax**: formal structure of **sentences**
  - **semantics**: **truth** of sentences wrt **models**
  - **entailment**: necessary truth of one sentence given another
  - **inference**: deriving sentences from other sentences
  - **soundess**: derivations produce only entailed sentences
  - **completeness**: derivations can produce all entailed sentences
- Wumpus world requires the ability to represent partial and negated information, reason by cases, etc.
- Forward, backward chaining are linear-time, complete for Horn clauses
- Resolution is complete for propositional logic
- Propositional logic lacks expressive power