Artificial Intelligence

Chapter 4, Sections 3-4

Stuart RUSSEL

reorganized by L. Aszalós

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Outline

- Hill-climbing
- Simulated annealing
- Genetic algorithms (briefly)
- Local search in continuous spaces (very briefly)

Iterative improvement algorithms

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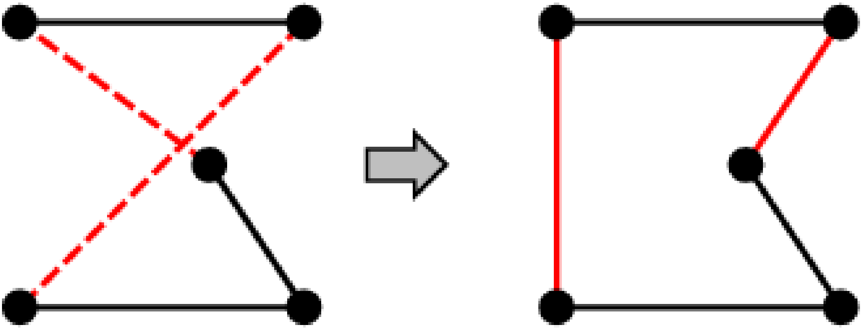
- Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

- Start with any complete tour, perform pairwise exchanges



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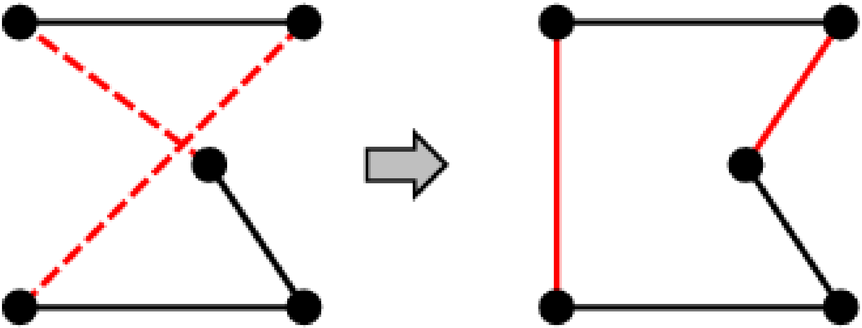

- Variants of this approach get within 1% of optimal very quickly with thousands of cities

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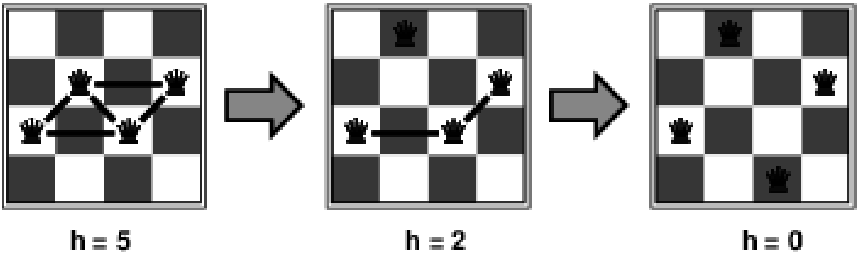

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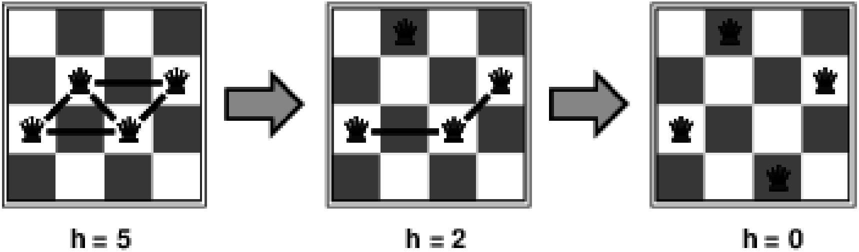


- Almost always solves n-queens problems almost instantaneously for very large n, e.g., $n = 1$ million

Hill-climbing (or gradient ascent/descent)

"Like climbing Everest in thick fog with amnesia"

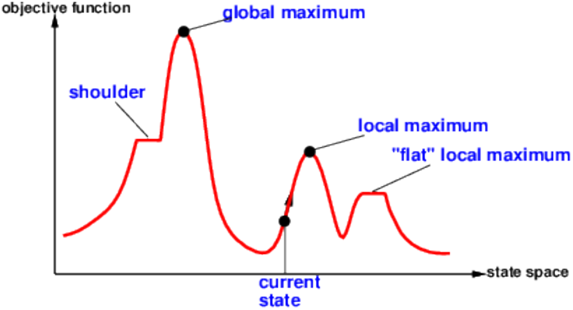
```
function Hill-Climbing(problem)
  returns a state that is a local maximum

  current: a node
  neighbor: a node

  current := Make-Node(Initial-State[problem])
  loop do
    neighbor := a highest-valued successor of current
    if Value[neighbor] <= Value[current]
      then return State[current]
    current:=neighbor
  end
```

Hill-climbing contd.

- Useful to consider **state space landscape**



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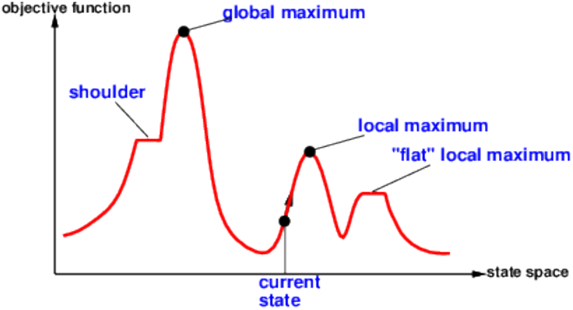

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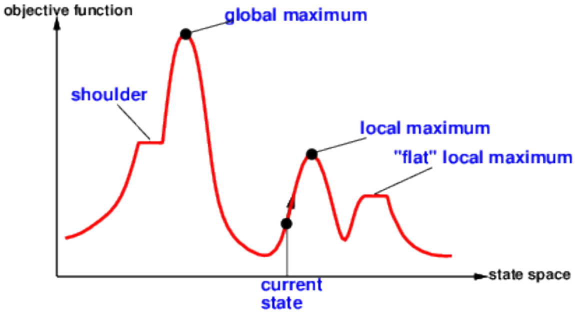

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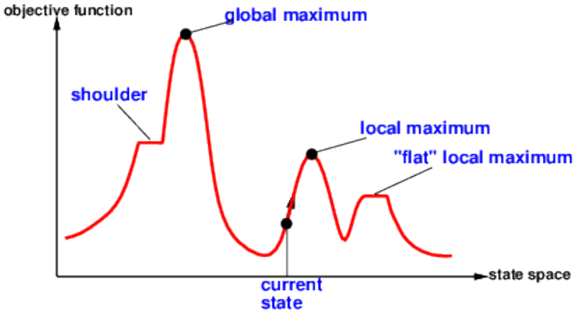

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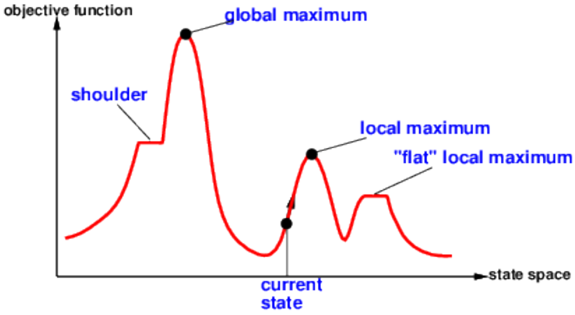

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- **Random sideways moves**
  - escape from shoulders :-)
  - loop on flat maxima :-(

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves *but gradually decrease their size and frequency*

```
function Simulated-Annealing(problem, schedule)
  returns a solution state

  schedule: a mapping from time to ``temperature''

  current: a node
  next: a node
  T: ``temperature'' controlling prob. of downward steps

  current := Make-Node(Initial-State[problem])
  for t=1 to infinity do
    T = schedule[t]
    if T=0 then return current
    next := a randomly selected successor of current
    Delta_E := Value[next]- Value[current]
    if Delta_E > 0 then current := next
    else current:= next, only with probability exp(Delta_E/T)
```

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- Widely used in VLSI layout, airline scheduling, etc.

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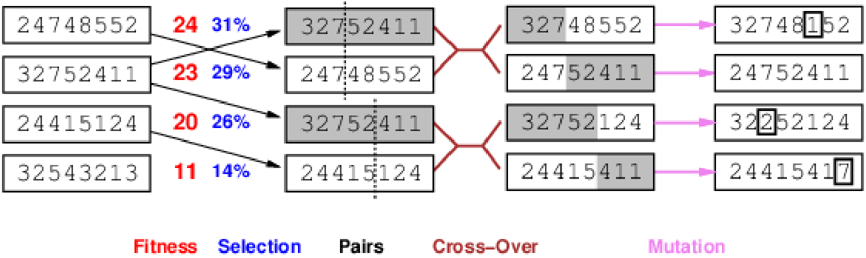
- Observe the close analogy to natural selection!

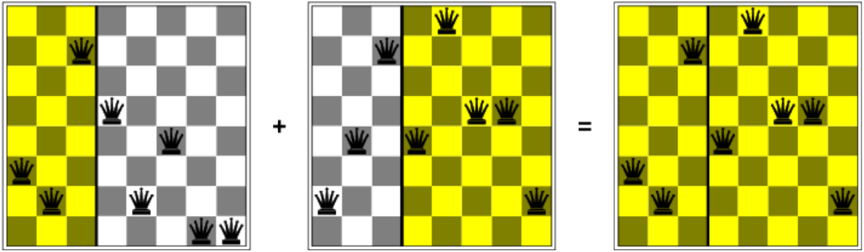
Genetic algorithms

- stochastic local beam search + generate successors from *pairs* of states



Genetic algorithms contd.

- GAs require states encoded as strings (**GPs** use **programs**)
- Crossover helps *iff substrings are meaningful components*



GAs $\neq$ evolution: e.g., real genes encode replication machinery!

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