Artificial Intelligence

Chapter 18: Learning from Observations

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reorganized by L. Aszalós

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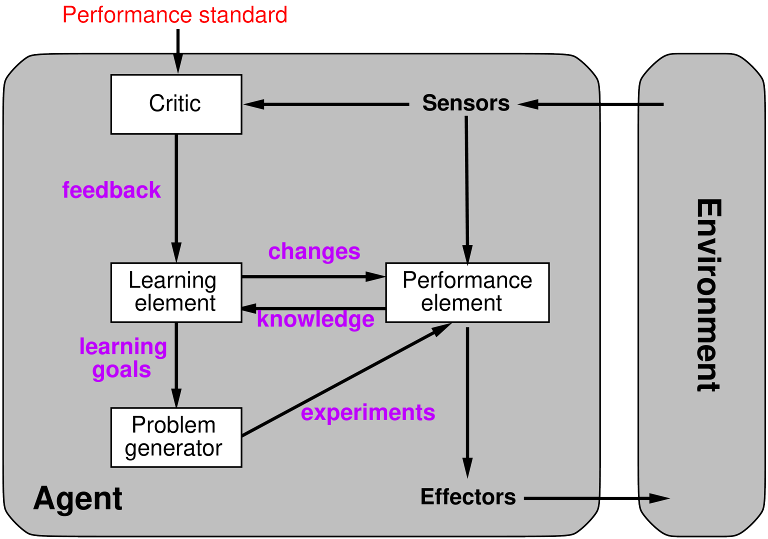
Outline

- Learning agents
- Inductive learning
- Decision tree learning
- Measuring learning performance

Learning

- Learning is essential for unknown environments,
  - i.e., when designer lacks omniscience
- Learning is useful as a system construction method,
  - i.e., expose the agent to reality rather than trying to write it down
- Learning modifies the agent's decision mechanisms to improve performance

Learning agents



Learning element

Design of learning element is dictated by

- what type of performance element is used
- which functional component is to be learned
- how that functional compoent is represented
- what kind of feedback is available

Example scenarios:

Performance element	Component	Representation	Feedback
Alpha–beta search	Eval. fn.	Weighted linear function	Win/loss
Logical agent	Transition model	Successor–state axioms	Outcome
Utility–based agent	Transition model	Dynamic Bayes net	Outcome
Simple reflex agent	Percept–action fn	Neural net	Correct action

- **Supervised learning**: correct answers for each instance
- **Reinforcement learning**: occasional rewards

Inductive learning (a.k.a. Science)

Simplest form: learn a function from examples (*tabula rasa*)

f is the **target function**

An **example** is a pair x-$f(x)$, e.g.,

$$\begin{array}{c|c|c} O & O & X \\ \hline & X & \\ \hline X & & \end{array}, \quad +1$$

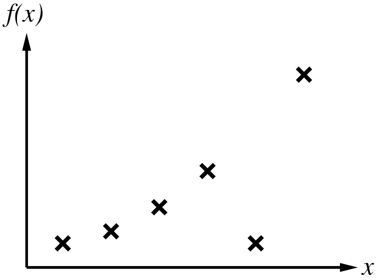
Problem: find a(n) **hypothesis** h such that $h \approx f$ given a **training set** of examples

This is a highly simplified model of real learning:

- Ignores prior knowledge
- Assumes a deterministic, observable "environment"
- Assumes examples are *given*
- Assumes that the agent *wants* to learn f—why?

Inductive learning method

Construct/adjust h to agree with f on training set (h is **consistent** if it agrees with f on all examples)
E.g., curve fitting:



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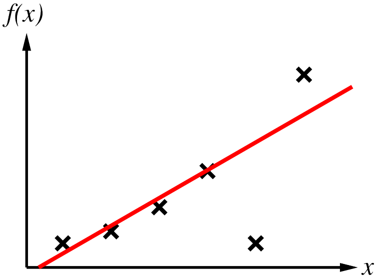

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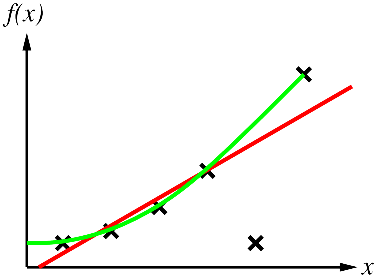

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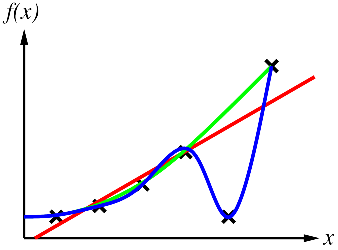

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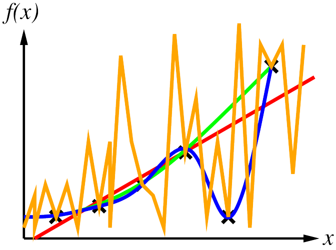

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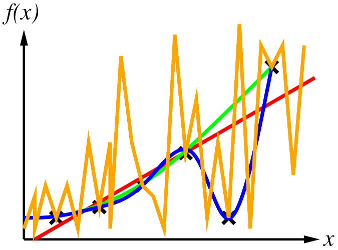


Ockham's razor: maximize a combination of consistency and simplicity

Attribute-based representations

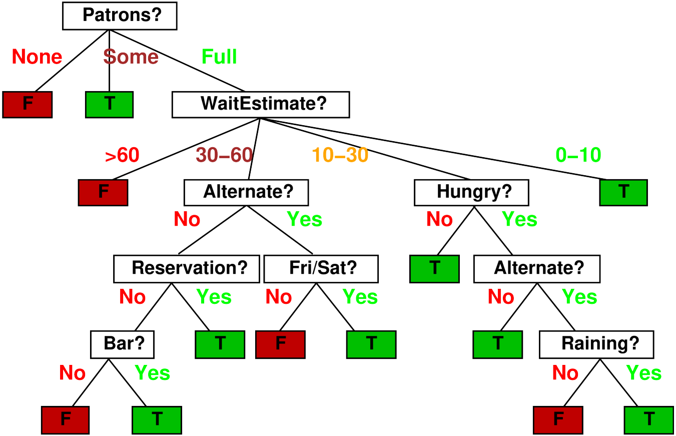
- Examples described by **attribute values**
  - (Boolean, discrete, continuous, etc.)
- E.g., situations where I will/won't wait for a table:

Example	Attributes										Target
	Alt	*Bar*	*Fri*	*Hun*	*Pat*	*Price*	*Rain*	*Res*	*Type*	*Est*	*WillWait*
X_1	*T*	*F*	*F*	*T*	*Some*	*$$$*	*F*	*T*	*French*	*0–10*	*T*
X_2	*T*	*F*	*F*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*30–60*	*F*
X_3	*F*	*T*	*F*	*F*	*Some*	*$*	*F*	*F*	*Burger*	*0–10*	*T*
X_4	*T*	*F*	*T*	*T*	*Full*	*$*	*F*	*F*	*Thai*	*10–30*	*T*
X_5	*T*	*F*	*T*	*F*	*Full*	*$$$*	*F*	*T*	*French*	*>60*	*F*
X_6	*F*	*T*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Italian*	*0–10*	*T*
X_7	*F*	*T*	*F*	*F*	*None*	*$*	*T*	*F*	*Burger*	*0–10*	*F*
X_8	*F*	*F*	*F*	*T*	*Some*	*$$*	*T*	*T*	*Thai*	*0–10*	*T*
X_9	*F*	*T*	*T*	*F*	*Full*	*$*	*T*	*F*	*Burger*	*>60*	*F*
X_{10}	*T*	*T*	*T*	*T*	*Full*	*$$$*	*F*	*T*	*Italian*	*10–30*	*F*
X_{11}	*F*	*F*	*F*	*F*	*None*	*$*	*F*	*F*	*Thai*	*0–10*	*F*
X_{12}	*T*	*T*	*T*	*T*	*Full*	*$*	*F*	*F*	*Burger*	*30–60*	*T*

- **Classification** of examples is **positive** (T) or **negative** (F)

Decision trees

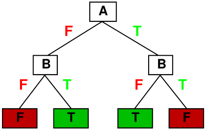
- One possible representation for hypotheses
- E.g., here is the "true" tree for deciding whether to wait:



Expressiveness

- Decision trees can express any function of the input attributes.
  - E.g., for Boolean functions, truth table row → path to leaf:

A	B	A xor B
F	F	F
F	T	T
T	F	T
T	T	F



- Trivially, there is a consistent decision tree for any training set
  - w/ one path to leaf for each example (unless f nondeterministic in x)
  - but it probably won't generalize to new examples
- Prefer to find more *compact* decision trees

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- How many distinct decision trees with n Boolean attributes?

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- How many purely conjunctive hypotheses (e.g., $Hungry \land \neg Rain$)?

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  - E.g., with 6 Boolean attributes, there are 18,446,744,073,709,551,616 trees
- How many purely conjunctive hypotheses (e.g., $Hungry \wedge \neg Rain$)?
  - Each attribute can be in (positive), in (negative), or out $\implies 3^n$ distinct conjunctive hypotheses
- More expressive hypothesis space
  - increases chance that target function can be expressed :-)
  - increases number of hypotheses consistent w/ training set
  - $\implies$ may get worse predictions :-(

Decision tree learning

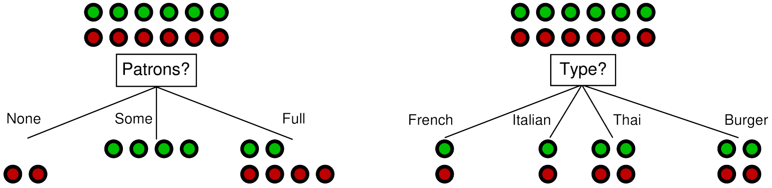
Aim: find a small tree consistent with the training examples
Idea: (recursively) choose "most significant" attribute as root of (sub)tree

```
func DTL(examples, attributes, default) → a decision tree

if examples  is empty then return default
else if all examples have the same classification then
        return the classification
else if attributes is empty then return Mode(examples)
     else best:= Choose-Attribute(attributes,examples)
          tree:= a new decision tree with root test best
          foreach value v_i of best do
              examples_i:= {elements of examples with best=v_i}
              subtree   := DTL(examples_i,attributes,best,Mode(examples))
                add a branch to tree with label v_i and subtree subtree
     return tree
```

Choosing an attribute

Idea: a good attribute splits the examples into subsets that are (ideally) *all positive* or *all negative*}



Patrons? is a better choice—gives *information* about the classification

Information

Information answers questions

The more clueless I am about the answer initially, the more information is contained in the answer

Scale: 1 bit = answer to Boolean question with prior $0.5, 0.5$

Information in an answer when prior is $P_1, \ldots, P_n$ is

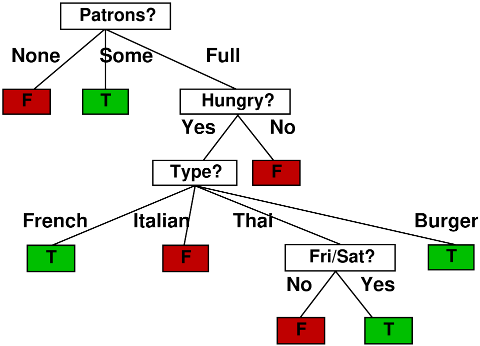
$H(P_1, \ldots, P_n) = \sum_{i=1}^{n} -P_i \log_2 P_i$ (also called **entropy** of the prior)

Information contd.

- Suppose we have p positive and n negative examples at the root
  - $\implies H(p/(p+n), n/(p+n))$ bits needed to classify a new example
- E.g., for 12 restaurant examples, $p = n = 6$ so we need 1 bit
- An attribute splits the examples E into subsets E_i, each of which (we hope) needs less information to complete the classification
- Let E_i have p_i positive and n_i negative examples
  - $\implies H(p_i/(p_i+n_i), n_i/(p_i+n_i))$ bits needed to classify a new example
  - $\implies$ *expected* number of bits per example over all branches is
  - $\sum_i \frac{p_i+n_i}{p+n} H(p_i/(p_i+n_i), n_i/(p_i+n_i))$
- For *Patrons*?, this is 0.459 bits, for *Type* this is (still) 1 bit
  - $\implies$ choose the attribute that minimizes the remaining information needed

Example contd.

Decision tree learned from the 12 examples:

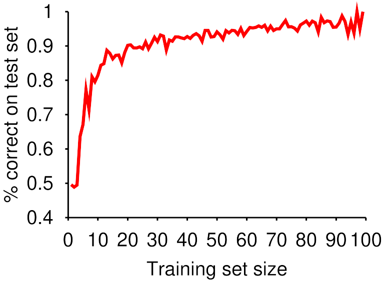


Substantially simpler than "true" tree—a more complex hypothesis isn't justified by small amount of data

Performance measurement

How do we know that $h \approx f$? (Hume's *Problem of Induction*)

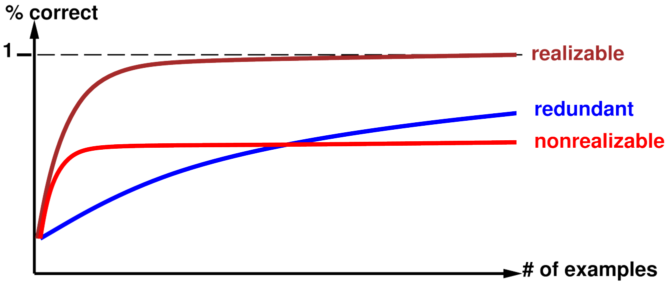
1. Use theorems of computational/statistical learning theory
2. Try h on a new **test set** of examples (use *same distribution over example space* as training set)



Performance measurement contd.

Learning curve depends on

- **realizable** (can express target function) vs. **non-realizable**
  - non-realizability can be due to missing attributes
  - or restricted hypothesis class (e.g., thresholded linear function)
- redundant expressiveness (e.g., loads of irrelevant attributes)



Summary

- Learning needed for unknown environments, lazy designers
- Learning agent = performance element + learning element
- Learning method depends on type of performance element, available feedback, type of component to be improved, and its representation
- For supervised learning, the aim is to find a simple hypothesis that is approximately consistent with training examples
- Decision tree learning using information gain
- Learning performance = prediction accuracy measured on test set