Logic/Switching CIRCUITS

GROUP ACTIVITY

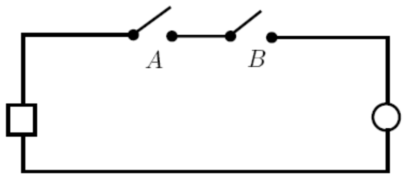
THEORETICAL CONTENTS

The following sections present the theoretical contents necessary to complete this activity.

1. Electronic circuits

The two positions of a switch are on and off, open (left) and closed (right):

Switches can be integrated as part of a circuit (being the square a battery and the circle a lamp).



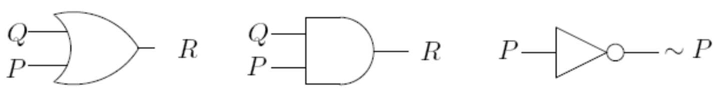
We can build a table depending on the state of the switches:

A	B	Lamp
Closed	Closed	On
Closed	Open	Off
Open	Closed	Off
Open	Open	Off

We can observe the analogy with truth tables if we substitute closed/on for true and open/off for false. It is easy to observe that this understanding of the truth table of this circuit matches the $A \wedge B$ truth table. We can conclude that two series-wired switches can be understood as a disjunction.

2. Digital logical circuits

The basic electronic components of a digital system are called digital logical circuits. The logical word indicates the role of logical mathematics in their development, and the digital term references the processing of discrete signals by the circuits. Instead of the T symbol for true and the F symbol for false, digital electronics use 1 and 0, respectively. The design of a logical circuit is achieved through the so-called logical doors: OR (left), AND (middle), and NOT (right). These doors consist of an input and an output, and depending on the values introduced as input, they yield different output values.



Each of these logical doors have an associated truth table that indicates the value of the inputs and outputs, matching the respective logical operators:

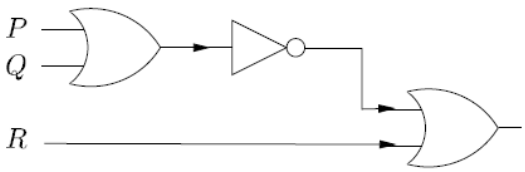
		NOT	*OR*	*AND*
P	Q	$R = \neg P$	$R = P \vee Q$	$R = P \wedge Q$
1	1	0	1	1
1	0	0	1	0
0	1	1	1	0
0	0	1	0	0

Combinational circuits are combinations of the logical AND, OR, and NOT doors, following a series of building rules:

1. An output can be used as input.
2. Two outputs cannot be combined, except through the inputs of a logical door.
3. An output can be divided into two in order to use it as input in two different logical doors.
4. Outputs of a logical door cannot be an input for that same logical door.

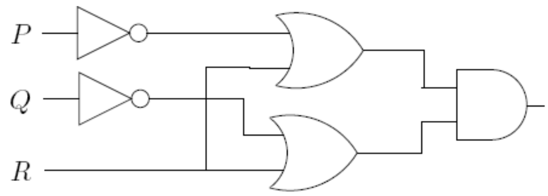
There are more complex circuits called ***sequential circuits***. Their construction follows the same rules, except for rule 4 (in sequential circuits, an output of a logical door can become a part of the input of that same door).

Combinational circuits can be assigned with a truth table and a matching WFF:



P	Q	R	$P \vee Q$	$\neg(P \vee Q)$	$\neg(P \vee Q) \vee R$
1	1	1	1	0	1
1	1	0	1	0	0
1	0	1	1	0	1
1	0	0	1	0	0
0	1	1	1	0	1
0	1	0	1	0	0
0	0	1	0	1	1
0	0	0	0	1	1

Two combinational circuits are equivalent if the WFFs that correspond to them are logically equivalent. For instance, the following circuit represents WFF $(\neg P \vee R) \wedge (\neg Q \vee R)$:



Through the logical equivalence chain $(\neg P \vee R) \wedge (\neg Q \vee R) \Leftrightarrow (\neg P \wedge \neg Q) \vee R \Leftrightarrow \neg(P \vee Q) \vee R$ we can verify that it is equivalent to the prior circuit.

The fundamental goals in digital circuit treatments are:

1. To build logical circuits that represent different WFFs or truth tables:
    a. If the circuit is specified through a WFF, it is enough to express it through conjunctions, disjunctions, and negations through the appropriate logical equivalences.
    b. If the circuit is specified through a truth table, we might leverage the conjunctive and disjunctive normal forms, since through them we can obtain WFFs composed only by disjunctions and conjunctions.
2. To choose the minimum cost circuit (that is, the circuit that is composed by the minimum possible number of logical doors) from several equivalent circuits that might represent the same WFF.
    a. It is necessary to use the appropriate logical equivalences to find the equivalent WFFs that use as little logical connectors as possible.
    b. This process is usually systematized by techniques such as Karnaugh's maps.

Do it yourself

1. Paint an electronic circuit with two switches in parallel, a battery, and a lamp.
    a. Build its truth table for 1=T/closed, 0=F/open
    b. Does it match the truth table of any logical operators?
2. What do you know about Karnaugh's maps and Veitch's diagrams?
    a. Karnaugh's maps for the simplification of Boolean functions:
        i. https://www.geeksforgeeks.org/introduction-of-k-map-karnaugh-map/
        ii. https://www.javatpoint.com/simplification-of-boolean-expressions-using-karnaugh-map

Solve:

	A	B	C	F	
0	0	0	0	0	
1	0	0	1	0	
2	0	1	0	1	$\to \bar{A}\,B\,\bar{C}$
3	0	1	1	1	$\to \bar{A}\,B\,C$
4	1	0	0	1	$\to A\,\bar{B}\,\bar{C}$
5	1	0	1	1	$\to A\,\bar{B}\,C$
6	1	1	0	1	$\to A\,B\,\bar{C}$
7	1	1	1	1	$\to A\,B\,C$

www.unicrom.com

Using the conjunctive and disjunctive normal forms, we have that:

$$(\neg A\wedge B\wedge\neg C)\vee(\neg A\wedge B\wedge C)\vee(A\wedge\neg B\wedge\neg C)\vee(A\wedge\neg B\wedge\neg C)\vee(A\wedge\neg B\wedge C)\vee(A\wedge B\wedge C)\Leftrightarrow(A\vee B\vee C)\wedge(A\vee B\vee\neg C)$$

www.unicrom.com

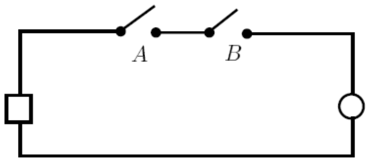
A \ BC	00	01	11	10
0	0	0	1	1
1	1	1	1	1

$$(A\vee B\vee C)\wedge(A\vee B\vee\neg C)\xLeftrightarrow[distributive]{}(A\vee B)\vee(C\wedge\neg C)\xLeftrightarrow[\substack{P\,\vee\,contradiction=true\,\leftrightarrow\,P=true\\ substitution}]{(C\wedge\neg C)\text{ is a contradiction}}(A\vee B)$$

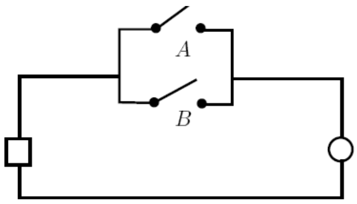
Exercises

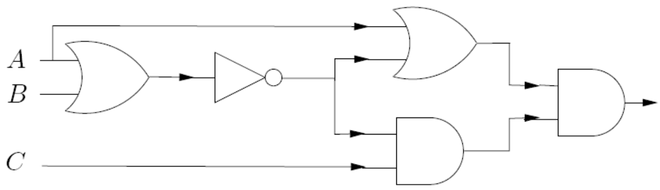
Exercise 1. Build the truth table for the following electronic circuits and indicate to which logical operator do they correspond. Squares are batteries and circles are lamps. Note: use the terms closed/on for true, and open/off for false.

a)



b)





a) Reason whether it is a combinatory circuit.

c) Add an element to the circuit to turn it into a sequential circuit.

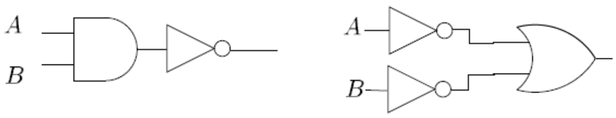
Exercise 3. Given the following Boolean expression: $(P \wedge (\neg Q \vee R)) \wedge Q$

a) Determine the corresponding combinational circuit.

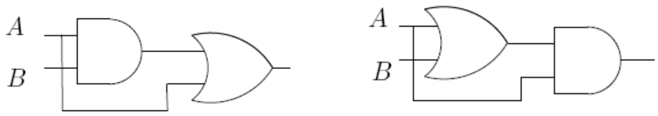
b) Write the input/output table associated to the circuit.

Exercise 4. Reason whether the following logical circuits are equivalent.

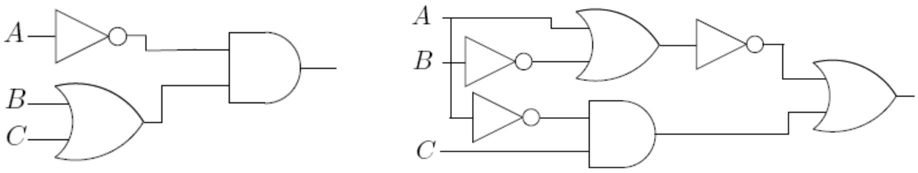
a)



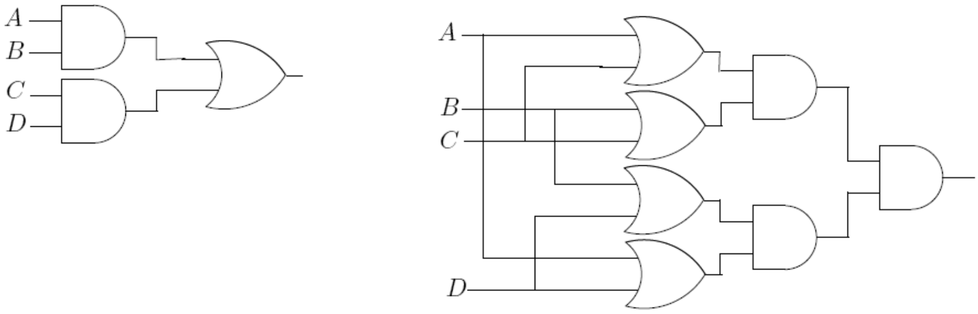
b)



c)



d)



References

Herbert B. Enderton, "A mathematical introduction to logic". Elsevier, 2001

W. K. Grassmann and J.P. Tremblay, "*Logic and discrete mathematics: a computer science perspective".* Prentice Hall, 1996.