

## Block 2. Probability

### PROPOSED PROBLEMS

**Problem 2.1.** You write the five vowels at random. What is the probability that the first vowel you write is “e” and the last one “o”?

A/  $1/20$

**Problem 2.2.** A box contains 15 white balls and 12 black balls. You draw two balls and do not return them to the box. What is the probability that you draw two black balls?

A/  $22/117$

**Problem 4.3.** A box contains 12 white balls and 8 black balls. If you draw two balls at random, what is the probability that both are the same colour?

A/  $47/95$

**Problem 2.4.** A box contains 12 white balls and 8 black balls. What is the probability that you draw two black balls if you return the first ball to the box?

A/  $4/25$

**Problem 2.5.** If the probability that a given event occurs is  $1/3$ , what is the probability that the event actually occurs if you make 4 tests?

A/  $65/81$

**Problem 2.6.** You draw two cards from a deck of 40 cards.

- a. What is the probability that the drawn cards are a knight and a three if you return the first card to the deck?

- b. What is the probability that the drawn cards are a knight and a three if you do not return the first card to the deck?

**A/** a.  $1/50$  b.  $4/195$

**Problem 2.7.** A box contains 8 white balls, 5 black balls, and 2 red balls. You draw three balls at random and you want to know:

- a. The probability that the three balls are white.  
b. The probability that two balls are white and one is black.

**A/** a.  $8/65$  b.  $4/13$ .

**Problem 2.8.** You draw 3 cards from a deck of 40 cards. Calculate the probability that:

- a. The three cards are knaves.  
b. They are an ace, a two, and a three.  
c. You draw a king, then a five, and then a seven.

**A/** a.  $1/2470$  b.  $8/1235$  c.  $4/3705$

**Problem 2.9.** A box contains two white balls and three black balls; another box has six white balls and four black balls. We draw a ball from each box. What is the probability that both balls are black?

**A/**  $6/25$

**Problem 2.10.** Using the numbers 1, 2, 3, 4, and 5, you write all possible three digit combinations, without repeating any digit in the same combination. If you choose one of the three digit combinations at random:

- a. What is the probability that it is a multiple of 4?  
b. And a multiple of 3?

**A/** a.  $1/5$  b.  $2/5$

**Problem 2.11.** A box contains 8 red balls, 4 blue balls, and 6 green balls. You draw 3 balls at random and you want to know:

- The probability that the three balls are red.
- The probability that two balls are red and one is green.
- The probability that two balls are blue and one is a different colour.
- The probability that all three are a different colour.
- The probability that all are the same colour.

**A/** a.  $7/102$  b.  $7/34$  c.  $7/68$  d.  $4/17$  e.  $5/51$

**Problem 2.12.** You roll a die 6 times. What is the probability that you get a 1 in 6 rolls?

**A/**  $31031/46656$

**Problem 2.13.** A box contains 2 white balls, 3 black balls, and 4 red balls. Another box contains 3 white balls, 5 black balls, and 4 red balls. You draw a ball at random from each box. What is the probability that both balls are the same colour?

**A/**  $37/108$

**Problem 2.14.** The probability that you hit a target in one shot is estimated to be 0.2. The probability that you hit the target in two shots is either  $P_1 = 0.04$ ;  $P_2 = 0.36$ ; or  $P_3 = 0.12$ . Determine which is the right answer.

**A/** 0.36

**Problem 2.15.** What is the probability of successfully hitting a ship if you can only launch three torpedoes and the probability of success of a single torpedo is estimated to be 30%?

**A/** 0.657

**Problem 2.16.** Consider the random experiment “roll a dice twice”. What is the probability that you get an even number in the second roll, given the condition that you must get an odd number in the first roll? Are these events dependent or independent? Why?

A/  $\frac{1}{2}$  independent

**Problem 2.17.** There are 8 red balls, 10 black balls, and 6 white balls in a bag. Three children draw, successively, two balls each, without returning any balls to the bag. Find the probability that the first child draws two red balls, the second draws two black balls, and the third one draws two white balls.

A/  $\frac{15}{9614}$

**Problem 2.18.** You roll a die “ $n$ ” times. What is the probability that you get a 6 in “ $n$ ” rolls?

A/  $1 - \left(\frac{5}{6}\right)^n$

**Problem 2.19.** In a random experiment, four coins are tossed in succession and you are asked:

- The probability that you get three tails at the most.
- The probability that you get two heads.

A/ a.  $\frac{15}{16}$  b.  $\frac{3}{8}$

**Problem 2.20.** A gun has 7 shells to hit a target. In each shot, the probability of hitting the target is  $\frac{1}{7}$ . What is the probability of hitting the target in 7 shots?

A/  $\frac{543607}{823543}$

**Problem 2.21.** The probability of a man living more than 25 years is  $\frac{3}{5}$ , whereas for a woman it is  $\frac{2}{3}$ . You are asked:

- The probability that both live more than 25 years.

- b. The probability that only the man lives more than 25 years.
- c. The probability that the woman lives more than 25 years.
- d. The probability that at least one of them lives more than 25 years.

**A/** a.  $2/5$     b.  $1/5$     c.  $4/15$     d.  $13/15$

**Problem 2.22.** If you draw 4 cards at random from a deck of 40 cards, determine:

- a. The probability that you draw two kings.
- b. The probability that three of the cards are the same suit.
- c. The probability that all the numbers are less than seven.

**A/** a. 0.04    b. 0.16    c. 0.12

**Problem 2.23.** You toss three coins in succession and consider the following events:

$A =$  "getting tails in the first toss".

$B =$  "getting any heads".

$C =$  "getting two heads".

You want to know:

- a. If  $A$  and  $B$  are incompatible.
- b. If  $A$  and  $B$  are independent.
- c. If  $A$  and  $C$  are incompatible.
- d. If  $A$  and  $C$  are independent

**A/** a. they are not incompatible    b. they are not independent    c. they are not incompatible  
d. they are not independent.

**Problem 2.24.** Among 100 people that attend a congress, 40 speak French, 40 speak English, 51 speak Spanish, 11 speak French and English, 12 speak French and Spanish, and 13 speak English and Spanish. You choose two attendees at random and you want to know:

- a. What is the probability that none of them speaks French?

- b. What is the probability that both speak Spanish?
- c. What is the probability that they understand each other only in Spanish?
- d. What is the probability that they only speak one language?
- e. What is the probability that they speak all the three languages?

**A/** a.  $\approx 0.3576$  b.  $\approx 0.2576$  c.  $\approx 0.0969$  d.  $\approx 0.5457$  e.  $\approx 0.0020$

**Problem 2.25.** A die is rigged, so that the probability of rolling a number is proportional to that number. Find the probability of getting an even number in one roll.

**A/** 4/7

**Problem 2.26.** In a survey of 24 students, 18 smoke cigarettes of the brand Ducados, 12 smoke cigarettes of the brand Celtas, and 8 smoke cigarettes of both brands. Three students are chosen at random and you want to know:

- a. What is the probability that the three of them smoke?
- b. What is the probability that exactly two of them smoke Ducados cigarettes?

**A/** a. 35/46 b. 459/1012

**Problem 2.27.** If out of 800 parts manufactured by a machine 25 of them were defective and you choose 5 parts at random, what is the probability of choosing any number of defective parts?

**A/** 0.1471

**Problem 2.28.** In a hospital specialised in respiratory diseases, 50% of admitted patients have bronchitis, 30% have pneumonia, and 20% have the flue. The probability of a total recovery from each of these diseases is 0.7, 0.8, and 0.9, respectively. One inpatient was discharged fully recovered. Find the probability that the patient had bronchitis.

**A/** 0.455

**Problem 2.29.** There is a cholera epidemic. One main symptom is diarrhoea, but this symptom also appears in people with food poisoning, and also in people who do not have a serious illness. The probability of having diarrhoea with cholera, food poisoning, or nothing serious is 0.99, 0.5, and 0.004, respectively. On the other hand, you know that 2% of the population has cholera, 0.5% has food poisoning, and the rest (97.5%) does not have a serious illness. You want to know:

- a. If you choose an individual from the population, what is the probability that this person has diarrhoea?
- b. If you know that a given individual has diarrhoea, what is the probability that this individual has cholera?

**A/** a. 0.0262 b. 0.0756

**Problem 2.30.** The probability that an item comes from factory  $A_1$  is 0.7, and from factory  $A_2$  is 0.3. You know that factory  $A_1$  produces 4 defective items per mil, and that factory  $A_2$  produces 8 per mil.

- a. An item is tested and found to be defective. What is the probability that it was produced in factory  $A_2$ ?
- b. An item is ordered from one of the factories at random. What is the probability that it is defective?
- c. 5 items are ordered from factory  $A_1$ . What is the probability that any of the items is defective?

**A/** a. 0.462 b. 0.0052 c. 0.99999

**Problem 2.31.** There is an epidemic in a population of animals. 10% of males and 18% of females are ill. Moreover, you know that there is twice as much females as males in the population:

- a. If you choose an individual from that population at random, what is the probability that it is ill?

- b. An individual from the population is known to be ill. What is the probability that it is male?

**A/** a. 0.153 b. 0.218

**Problem 2.32.** In a classroom, there are 30 female students, 15 students repeating the school year, 10 of them male students, and there are 15 male students who are not repeating the school year. You are asked:

- How many students are in the classroom?
- If you choose a student at random, what is the probability that he is male?
- If you choose a student at random, what is the probability that she is female and is repeating the school year?
- If you choose two students at random, what is the probability that none of them is repeating the school year?

**A/** a. 55 (suggestion: use a contingency table to obtain the total) b.  $5/11$  c.  $1/11$  d.  $52/99$ .

**Problem 2.33.** The probability that a student passes Mathematics is 0.6, that he passes English Language is 0.5, and that he passes both subjects is 0.2. Find:

- The probability that he passes at least one of the subjects.
- The probability that he does not pass any of the subjects.
- The probability that he passes Mathematics, but he does not pass English.

**A/** a. 0.9 b. 0.1 c. 0.4.