Advanced Calculus

Topology: Definition

ThinkBS: Basic Sciences in Engineering Education

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What is a Topology on a set?

Assume that X is a non-empty set. A collection $\mathcal{T} \subseteq P(X)$ of subsets of X is called a topology on X, if

1. $\emptyset, X \in \mathcal{T}$
2. Any finite intersection of elements of $\mathcal{T}$, belongs to $\mathcal{T}$.
3. Any arbitrary union of elements of $\mathcal{T}$, belongs to $\mathcal{T}$.

A set together with a topology on it is called a topologic space. An element of $\mathcal{T}$ is called an open set.

Example 1: $P(X)$ is a topology on any set X. It is called the discrete topology.

Example 2: $\{\emptyset, X\}$ is a topology on any set X. It is called the trivial or indiscrete topology.

Example 3: The collection of all open sets in a metric space, is a topology on the underlying set. It is called metric topology.

Inherited Topology

Assume that $(X, \mathcal{T})$ is a topological space and $Y \subseteq X$. In this case,

$$\mathcal{T}_{|Y} = \{O \cap Y \mid O \in \mathcal{T}\}$$

is a topology on Y, which is called subspace or inherited topology. (why $\mathcal{T}_{|Y}$ is a topology?)

The concept in terms of metric spaces is as follows: Assume $(Y, d_{|Y}) \subseteq (X, d)$ is obtained by restricting d to $Y \times Y$. In this case, a set $H \subseteq Y$ is open in $(Y, d_{|Y})$ iff there is an open set $G \subseteq X$ such that $H = G \cap Y$.