Abstract Algebra

Cyclic Groups

ThinkBS: Basic Sciences in Engineering Education

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Cyclic Groups

Let G be a group and let $g \in G$. Then

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is a subgroup of G and is the smallest subgroup of G that contains g. This subgroup is called the cyclic group generated by g.

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Example 2: $< 1 >=< 3 >= \mathbb{Z}_4$ is a cyclic group (additive notation).

Properties of Cyclic Groups

- Every cyclic group is Abelian.
- A subgroup of a cyclic group is cyclic.
- The subgroups of $\mathbb{Z}$ under addition are precisely the groups $n\mathbb{Z}$ under addition for $n \in \mathbb{Z}$.

For further properties of cyclic groups and their subgroups look at Part 1 Section 6 of the textbook.