Abstract Algebra

Subgroups

ThinkBS: Basic Sciences in Engineering Education

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Important Notation

From now on we will refer to the binary operation of the group '$*$' simply as '.' and groups identity element e will be shown by 1. In such notation inverse of an element g will be shown by g^{-1}.

This is called the 'multiplicative notation'. One should not misunderstand this 'abuse of notation' for normal multiplication of numbers.

When talking about Abelian groups, and also when we put more binary operations on a set (like in case of Rings and Fields), we will use 'additive notation': $+$ instead of $*$, 0 instead of e and $-g$ for inverse of g.

When clear from context what is the group operation, we will refer to a group by simply referring to its underlying set.

Important Notation

In a group G, for a $g \in G$ we define:

$$g^0 = 1$$
$$g^n = g.g^{n-1} \quad \textit{for} \quad n \geq 1$$
$$g^{-n} = (g^{-1})^n$$

Subgroups

A subset H of a group G is called a subgroup of G if it is closed under the binary operation of G and with the induced operation from G is itself a group. In such ase we write $H \leq G$ to show that H is a subgroup of G.

For instance: $2\mathbb{Z} \leq \mathbb{Z}, \quad \mathbb{Q} \leq \mathbb{R}, \quad (\mathbb{Q}^{>0}, .) < (\mathbb{R}^{>0}, .)$

$\{0\} \leq \{0, 2\} \leq \mathbb{Z}_4$

For further details look at Part 1 Section 5 of the textbook.