Abstract Algebra

Abelian Group, Finite/Infinite Group

ThinkBS: Basic Sciences in Engineering Education

Kadir Has University, Turkey

Abelian Groups

$(G, *)$ is called an Abelian group if for every element g, $g' \in G$ we have $g * g' = g' * g$.

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Consider the set $GL_2(\mathbb{R})$ of all 2×2 **invertible** matrices with real elements. This set together with matrix operation is a group but not an Abelian one (why?)

Finite and Infinite Groups

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$\mathbb{Z}_n$ with mod n multiplication is a group iff n is a prime number.

For further properties and examples of groups, look at Part 1 Section 4 of the textbook.

Group Tables

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Examples of such tables can be found in Part 1 Section 2 of the textbook.

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Construct the tables for groups with $n = 2$, 3, 4 and 5 elements. How many different 'choices' for each n you can find? Are all of these groups Abelian?