Abstract Algebra

Definition of Group

ThinkBS: Basic Sciences in Engineering Education

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Groups

A Group is a set G with a binary operation (usually shown as $*$) $* : G \times G \to G$ such that satisfies the followings axioms:

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3. For each element $g \in G$, there exists an element $g' \in G$ such that $g * g' = g' * g = e$ (inverse of g with respect to $*$)

Groups: notes on axioms

If a set G with a binary operation $*$ satisfies 'Group Axioms', then we normally show it as $(G, *)$.

- $(\mathbb{Z}, +)$, $(\mathbb{Q}, +)$, $(\mathbb{Q} \setminus \{0\}, .)$, $(\mathbb{R}, +)$ are all examples of groups.

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- Identity element of a group is unique (which we will show by 0 or 1 depending on the context).
- Inverse element of each element in a group is unique.