

Season 1: Exercises (PDE's analytical solution)

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Introduction to PDE's

1.
$$\begin{cases} U_{tt} = 4U_{xx} & 0 < x < \pi, \quad t > 0 \\ U(0, t) = U(\pi, t) = 0, & t > 0 \\ U(x, 0) = 0, & 0 \leq x \leq \pi \\ U_t(x, 0) = 2 \sin(4x) + \sin(6x), & 0 \leq x \leq \pi \end{cases}$$
2.
$$\begin{cases} U_{tt} = 4U_{xx} & 0 < x < \pi, \quad t > 0 \\ U(0, t) = U(\pi, t) = 0, & t > 0 \\ U(x, 0) = \sin(3x) - 4 \sin(10x), & 0 \leq x \leq \pi \\ U_t(x, 0) = 2 \sin(4x) + \sin(6x), & 0 \leq x \leq \pi \end{cases}$$
3.
$$\begin{cases} U_t = U_{xx} - 4U & 0 < x < \pi, \quad t > 0 \\ U(0, t) = U(\pi, t) = 0, & t > 0 \\ U(x, 0) = \sin(x) - 4 \sin(2x), & 0 \leq x \leq \pi \end{cases}$$
4.
$$\begin{cases} U_t = 7U_{xx} & 0 < x < \pi, \quad t > 0 \\ U(0, t) = U(\pi, t) = 0, & t > 0 \\ U(x, 0) = 3 \sin(2x) - 6 \sin(5x), & 0 \leq x \leq \pi \end{cases}$$
5.
$$\begin{cases} U_{tt} + 6U_t + 9U = U_{xx} & 0 < x < \pi, \quad t > 0 \\ U(0, t) = U(\pi, t) = 0, & t > 0 \\ U(x, 0) = 0 & 0 \leq x \leq \pi \\ U_t(x, 0) = 3 \sin(8x), & 0 \leq x \leq \pi \end{cases}$$
6.
$$\begin{cases} U_{tt} = U_{xx} & 0 < x < \pi, \quad t > 0 \\ U(0, t) = U(\pi, t) = 0, & t > 0 \\ U(x, 0) = \sin(2x) & 0 \leq x \leq \pi \\ U_t(x, 0) = \sin^3(x), & 0 \leq x \leq \pi \end{cases}$$