## ThinkBS: Basic Sciences in Engineering Education Erasmus Plus Project, Summer School - Turkey in blended learning format between 25 July - 05 August, 2022

ThinkBS is a European Project implemented in a partnership of four universities from Turkey, Spain, Romania and Hungary aiming to improve applied mathematics skills of students and graduates in the field of engineering sciences.

Between July, the $25^{\text {th }}$ and August, the $5^{\text {th }}, 2022$, courses and applications are presented, as follows:

| Date | Course | Lecturer |
| :---: | :---: | :---: |
| $\begin{gathered} \text { 25.07.2022 } \\ 10 \mathrm{am}-13 p m(G M T+3) \\ \hline \end{gathered}$ | Advanced Linear Algebra | Prof. Ayşe H. Bilge |
| $\begin{gathered} \text { 26.07.2022 } \\ 10 a m-13 p m(G M T+3) \end{gathered}$ | Advanced Linear Algebra | Prof. Ayşe H. Bilge |
| $\begin{gathered} \text { 27.07.2022 } \\ 10 a m-13 p m(G M T+3) \end{gathered}$ | Advanced Linear Algebra | Prof. Ayşe H. Bilge (Tutorials and consultancy for students) |
| $\begin{gathered} \text { 28.07.2022 } \\ 10 a m-13 p m(G M T+3) \end{gathered}$ | Biomathematics | Prof. Ángela Jimeno |
| $\begin{gathered} \text { 29.07.2022 } \\ 10 a m-13 p m(G M T+3) \end{gathered}$ | Biomathematics | Prof. Francisco José Roig |
| $\begin{gathered} \text { 01.08.2022 } \\ \text { 10am - 13pm }(G M T+3) \end{gathered}$ | Advanced Calculus, | Arash Rezaeinazhad |
| $\begin{gathered} \text { 02.08.2022 } \\ \text { 10am - 13pm }(G M T+3) \end{gathered}$ | Advanced Calculus, | Arash Rezaeinazhad |
| $\begin{gathered} \text { 03.08.2022 } \\ 10 \mathrm{am}-13 p m(G M T+3) \end{gathered}$ | Advanced Calculus | Arash Rezaeinazhad (Tutorials and consultancy for students) |
| $\begin{gathered} \text { 04.08.2022 } \\ 9 \mathrm{am}-13 p m(G M T+3) \end{gathered}$ | Abstract algebra | Prof. Ayşe H. Bilge |
| $\begin{gathered} \text { 05.08.2022 } \\ 9 a m-13 p m(G M T+3) \end{gathered}$ | Abstract algebra | Prof. Ayşe H. Bilge |




Participants will receive a Certificate of Attendance including the Summer School details and the Erasmus + European project context.

The modules target students, PhD students, specialist interested in mathematics applications in the fields of engineering and applied sciences. Projects and applications will be customized in line with the students level so as to acquire technical and transversal skills and promote teamwork.

Registration is open by email at: beyzakucukdag@stu.khas.edu.tr
Zoom link: https://us02web.zoom.us/i/6320098921
Meeting ID: 6320098921

## Lecture Descriptions:

## July 25, 2022 Session I - Advanced Linear Algebra

- We introduce the notion of "vector space" as a set of objects with certain operations.
- We give the set of polynomials and functions with standard addition operation as an example.
- We describe linear independence, bases and coordinates in this general setting.
- We describe linear operators, the matrix of a linear operator and we discuss how the differentiation of polynomials is represented as the multiplication of a matrix with a vector.


## July 26, 2022 Session II - Advanced Linear Algebra

- We introduce the notion of invertibility of a matrix and we discuss the solution of linear systems of equations in terms of matrix algebra;
- We define the determinant of a matrix and introduce the characteristic polynomial of a matrix and show that every matrix annihilates its own characteristic polynomial.
- We define the minimal polynomial of a matrix as the monic polynomial of lowest degree that is annihilated by the given matrix.
- We define similar matrices as the ones that represent the same linear transformation in terms of different bases. We discuss how the characteristic and minimal polynomials are used in the determination of similarity classes.




July 27, 2022 Session III - Advanced Linear Algebra

- We discuss the "diagonalizability" of linear operators. We define functions of matrices, in particular we discuss how one can define the exponential of a matrix.
- We introduce "inner product spaces" and generalize the notion of orthogonality in abstract linear spaces.
- We discuss complete orthonormal sets and their applications in Fourier series.

July 28, 2022 Session I - Introduction to Biomathematics

- Descriptive statistics and exploratory data analysis
- Hypothesis testing
- Correlation, regression and clustering methods

July 29, 2022 Session II - Population Genetics and Population Dynamics in Biomathematics.

- Description of Population Genetics
- Allelic an Genotypic Frequency
- Hardy Weinberg Equilibrium
- Population Genetics as Forensic tool

August 1, 2022 Session I - Advanced Calculus

- We discuss the definition of rational numbers and explore the notions of countability and uncountability
- We discuss the definition of real numbers, order relations, and the "topology" of the real line
- We define metric spaces and discuss the "topology" of the Euclidean spaces as a metric space. We discuss the notions of connectedness, compactness and continuity in the setting of metric spaces.
- We discuss the notion of convergence for sequences of numbers and power series. We show how convergent series can be used to define new functions.


## August 2, 2022 Session II - Advanced Calculus

- We define "derivative" for functions of a single variable and discuss how this generalized to functions of several variables.
- We define the Riemann integral and discuss how to define new functions in terms of definite integrals.
- We define various notions of convergence for sequences of functions and discuss the difference between "pointwise convergence" and "uniform convergence".


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- We discuss the construction of the "Dirac's delta function" as a limit and its definition in terms of integrals.


## August 3, 2022 Session III - Advanced Calculus

- We define differentiation for complex functions, Cauchy Riemann equations, and complex analytic functions.
- We discuss the difference between infinitely differentiable functions and real analytic functions. We show how such functions can be used to define "local" objects, using "partitions of unity" and their use in the construction of local charts on "manifolds".
- We briefly describe differential forms together with their relation to integration and we unify, the Stokes and the divergence theorems in this setting.


## August 4, 2022 Session I - Abstract Algebra

- We define "groups" as a set with a single operation, and illustrate their properties using modular arithmetic and permutations of a finite set. We discuss the distinction between abelian and non-abelian groups
- We discuss subgroups, normal subgroups and quotient groups.
- We define "rings" as a set with two compatible operations and discuss notion of "zero divisors". We illustrate these notions on matrices.
- We define "ideals" in rings and the "quotient ring"
- We discuss difficulties associated with defining a "product" of vectors, describe the "octonions" and "quaternions".


## August 5, 2022 Session II - Abstract Algebra

- We define "modules" as an analogue of "vector spaces".
- We define "fields" as a generalization of the real numbers with the usual addition and multiplication operations.
- We discuss the "field extension" as a generalization of the construction of complex numbers starting from real numbers.
- We discuss the notion of "algebraically closed" fields, and illustrate this notion on the factorization of polynomials over real and complex numbers.
- We briefly introduce "Galois theory" and show how this is related to the solvability of polynomials equations in terms of radicals.


