Advanced Calculus

Alternating Series, Absolute and Conditional Convergence

ThinkBS: Basic Sciences in Engineering Education

Kadir Has University, Turkey

Alternating Series

Let $\{a_n\}$ be a sequence of positive real numbers. The series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n$ is called an alternating series.

Theorem: (Leibniz's Test) Let $\{a_n\}$ be a decreasing sequence of positive real numbers, i.e. $a_1 \geq a_2 \geq a_3 \geq ...$ with $\lim_{n\to\infty} a_n = 0$. Then the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1}a_n = a_1 - a_2 + a_3 - a_4 + \ldots$ is convergent.

Example: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is convergent.

Absolute and Conditional Convergence

A series $\sum a_n$ of real numbers is called absolutely convergent if the series $\sum |a_n|$ converges. If $\sum a_n$ is convergent and $\sum |a_n|$ does not converge, then $\sum a_n$ is called conditionally convergent.

Example 1: Any absolutely convergent series is convergent. (why?)

Example 2: The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ is conditionally convergent. (why?)