Musical Set Theory

Mathematics is everywhere, and certainly between musical notes. Musical set theory is the application of mathematical set theory to study music. For this aim, the structures that can be put on the sets to make them into 'groups' are used rather than pure set theory itself.

Definition (Group) A non-empty set G together with a binary operation $\circ\colon G \times G \to G$, called 'addition' or 'multiplication' depending on context, is called a *Group*, and is shown by $(G, \circ)$, if the following properties are satisfied:

1. For every x, y, $z \in G$, $(x \circ y) \circ z = x \circ (y \circ z)$ (Associativity),
2. There exist an element $e \in G$ such that for all $x \in G$, $e \circ x = x \circ e = x$ (Identity element),
3. For every $x \in G$ there is a $y \in G$ such that $x \circ y = y \circ x = e$ (Inverse element),

- In addition to above conditions, if for a group G it happens that for all x, $y \in G$, $x \circ y = y \circ x$, then the group is called *Abelian*.

Now to go back to music theory, we assign a number to every pitch: 1 to *Do*, 2 to *Re* and so on till 7 is assigned to *Si*. The same numbers are assigned to the pitches with the same name on higher or lower intervals. In this way, one assigns the Set $\mathbb{Z}_7$ of integers modulo 7 to the pitches.

- Prove that $(\mathbb{Z}_7, \circ)$, where $\circ$ is addition modulo 7 is an Abelian Group.
- Prove that $(\mathbb{Z}_7 \setminus \{0\}, \circ)$, where this time $\circ$ is multiplication modulo 7 is an Abelian Group.

Please send your answers to "ayse.bilge@khas.edu.tr".

References

[1] Hungerford, T. W. (1980). Algebra, volume 73 of. Graduate Texts in Mathematics.