

Euler's Identity

We are going to find out about “the most beautiful equation in Mathematics”. The main reason for this title is that in this formula, the most known constants appear in a very simple form.

Definition (Complex Exponential) For a complex number z we define

$$e^z := \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

1. Replace z with a pure imaginary number ‘ ix ’ in the above formula and simplify the terms.
2. By regrouping if necessary, show that

$$e^{ix} = \cos(x) + i \sin(x)$$

3. Replace x by π and simplify. You should be able to see all of the five numbers $0, 1, \pi, e$ and i together.

Please send the most beautiful equation in Mathematics to “ayse.bilge@khas.edu.tr”.

References

- [1] Palka, B. P. (1991). An introduction to complex function theory. Springer Science & Business Media.