

## Algebraic versus Transcendental Numbers

**Definition** (Algebraic and Transcendental Number) A real number  $r$  is called an *algebraic* number if  $r$  is the root of a polynomial in one variable with rational coefficients. Otherwise it is called a *Transcendental* number.

- Prove that every rational number is an algebraic number.
- Prove that  $\sqrt{2}$  and  $\sqrt[3]{2} - \sqrt{3}$  are algebraic numbers. (This means that for each of these numbers you need to construct a polynomial in one variable with rational coefficients for which these given numbers are a root.)
- Prove that the set of algebraic numbers is a countable set.

Please send your answers to “ayse.bilge@khas.edu.tr”.

## References

- [1] Rudin, W. (1964). Principles of mathematical analysis (Vol. 3). New York: McGraw-hill.